ROBUST SENSITIVITY ANALYSIS OF COURSES OF ACTION USING AN ADDITIVE VALUE MODEL

THESIS

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Abstract

The Department of Defense (DoD) requires the ability to quantifiably measure progress in arenas that are complex and difficult to measure, such as the stability of a region. Therefore, the DoD works diligently to predict the effect of operations and sponsors research to improve prediction and analysis. They desire a repeatable, systematic methodology to aid in the selection of courses of action (COA) that efficiently meet stated objectives and quantitatively measure the degree of accomplishment of these objectives. The author proposes a value-focused thinking (VFT) decision analysis (DA) approach to this problem. This methodology not only aids in selection of possible COAs, but provides a framework to compare the effectiveness of implemented actions via key indicators. Due to the complex nature of COA selection and assessment, weights within the DA model are often fluid. Sensitivity analysis provides the justification of COA selection in such an environment. This thesis focuses on conducting further analysis of the ranked alternatives through a robust sensitivity analysis technique.

Sensitivity analysis begins with the examination of the top ranked alternative by varying one weight at a time, one-way sensitivity. The author then proposes a more robust examination of multiple weight sensitivity using five unique measures and optimization via linear and non-linear programming. The measures reveal the alternatives sensitive to small simultaneous variations of multiple weights within the model, n-way sensitivity. Small measure values indicate sensitive alternatives, and indicate to a field commander where to more closely examine the consequences of a selected COA.

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To Mother, Family, and Friends

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Hunter A. Marks

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ROBUST SENSITIVITY ANALYSIS OF COURSES OF ACTION USING AN ADDITIVE VALUE MODEL

I. Introduction

Increasingly, the military is using decision analysis techniques to support commanders in selecting Courses of Action (COAs). Many of these approaches assume additive linear weights. This thesis examines the decision sensitivity to variations of those weights. In particular, it examines the robustness of a selected solution with respect to multiple weight changes in an additive value model. This section examines the nature of military COA selection, current doctrine and planning processes, and planned enhancements. The extensiveness, timeliness, and fluidity of COA selection continue to challenge the military commander's ability to make informed decisions.

I.A. Background

A commander's desire to make decisions with conflicting objectives dates back at least as far as the sixth century B.C. to the days of Sun Tzu (Wu, 2004:397). According to Griffith's translation of Sun Tzu's The Art of War, a commander should, "…determine the enemy's plans and…know which strategy will be successful and which will not" (Tzu, 6th cent. B.C.:152). In 2002, the United States Air Force Scientific Advisory Board defined predictive battlespace awareness as the desire to thoroughly know and understand an adversary (SAB-TR-02-01, Vol. 1, 2002:2). A thorough understanding of the adversary includes the lofty goal of predicting the adversary's actions before they put them into motion. Limitation of this knowledge requires the commander's staff to develop feasible courses of action in an uncertain environment.

Current doctrine and planning processes strive to address the challenges presented by the nature of this leadership task. Joint doctrine provides the United States armed forces overarching guidance to achieve the President's and Secretary of Defense's strategic goals by bridging the gap between policy and employment. It describes war as, "…a complex, human undertaking that does not respond to deterministic rules" (JP 1, 2007:Sec. I, 1). An adversary in war plays by many sets of rules which do not often remain stagnant. Neither commanders nor their staffs fully know the intentions of an adversary, especially in today's information age and in the face of asymmetric warfare where targets and tactics change in a matter of minutes. This produces a capability gap between a commander's desire and capabilities the commander possesses.

Joint Publication 3-0, *Joint Operations*, defines a system as "a functionally related group of elements forming a complex whole" (JP 3-0, 2006:Sec. II, 21). This definition includes the complex interaction of political, military, economic, social, information, and infrastructure (PMESII) elements in which today's military must operate. The doctrine stresses this system perspective in order for planners to gain a better understanding of the interactions within and between friendly, adversarial, and neutral systems (JP 3-0, 2006:Sec. II, 22).

In addition to extensive system understanding, joint doctrine asks a Joint Force Commander (JFC) to employ unified action. Unified action consists of synergistically applying all instruments of national and multinational power (JP 3-0, 2006:Sec II, 3). Instruments of national power include diplomatic, information, military, economic, financial, intelligence, and law enforcement (DIMEFIL) actions at the disposal of a nation or coalition of nations. A JFC must integrate and synchronize the actions of joint

US military forces, multinational forces, intergovernmental organizations, nongovernmental organizations, and other US government agencies to achieve unified action (JP 3-0, 2006:Sec. II, 4).

Commanders desire the capability to continuously assess the progress towards achievement of their objectives in today's environment (JP 5-0, 2006:Sec III, 57). This capability expands a commander's solution space for selecting a feasible course of action (COA). A commander should assess the direct and indirect effects of a COA on an adversary's political, military, economic, social, information, and infrastructure (PMESII) systems (JP 3-0, 2006: Sec. IV, 9). COAs consisting of purely kinetic or purely military options may limit the ability to best achieve a commander's desired effect used to reach a certain end-state. Instead, one should evaluate feasible COAs consisting of all elements of national power to include diplomatic, information, military, economic, financial, intelligence, and/or law enforcement (DIMEFIL) options.

A 2004 Capabilities Development Document (CDD) for Air and Space Operations Center (AOC) as a Weapon System provided guidance for developing a toolset to enable dynamic, ongoing effect based assessment (EBA) (CDD AOC WS, 2006:np). Since the Air Force Research Laboratory (AFRL) has the mission of "leading the discovery, development, and integration of affordable warfighting technologies for America's aerospace forces" (AFRL PA, 2007:np), they are supporting this EBA requirement. The Air Force Research Laboratory/Information Directorate (AFRL/RI) develops systems, concepts, and technologies to enable the warfighter in today's challenging information age (AFRL PA, 2007:np). A current focus of AFRL/RI is the development of tools to support a commander, such as the Joint Force Commander (JFC) or Joint Forces Air

Component Commander (JFACC), in making more informed decisions when selecting courses of action (COAs). Two programs currently under development in response to this guidance include the Commander's Predictive Environment (CPE) and the Dynamic Air and Space Effects-base Assessment (DASEA).

One decision analysis approach considered to address the challenge of developing COAs in a rapidly evolving environment is value-focused thinking (VFT). This technique focuses on what is important (values) to the decision maker (DM) and elicits weighting of the values. The solicited values form a weighted value hierarchy to assist in the decision making process. This process attempts to aid the decision maker in the COA Development portion of the Joint Operation Planning Process (JOPP).

Inappropriate weights caused either by subjectivity or a rapidly changing situation could affect the decision. Conducting sensitivity analysis, by varying one or two weights from zero to one and changing the others proportionally to see how the decision changes, attempts to determine the range of weights for which the model recommends the same decision. One-way sensitivity analysis ignores the possible interaction between two or more weights in a hierarchy. Two-way sensitivity analysis addresses the interaction of two weights in the hierarchy, but ignores higher interactions. In a rapidly changing environment, such as combat, are the solicited weights appropriate, or in a feasible range where the decision remains the same? If the weights are not in a feasible range, do they affect the decision? Do interactions of more than three weights affect the COA recommended to the commander? This provides a method to address these questions.

I.B. Problem Statement

Commanders and their staffs face difficult situations requiring them to make decisions under conditions of uncertainty. Many quantitative methods elicit preferences from them as model inputs. Reaching a consensus on subjective preferences, often resulting in weights, sometimes proves difficult for a group decision (Ewing *et al*, 2006:41). While many traditional decision analysis methods analyze the sensitivity of a decision to changes in one factor or the interaction of two factors, the interactions of more than two factors could affect the outcome. The use of a decision model in an uncertain world requires it to be flexible and robust to these interactions.

I.C. Research Scope

This thesis uses value-focused thinking to generate and assess courses of action to achieve a commander's objectives. Preferably, the leading commander or his/her staff should provide the needed inputs to create the decision model. Sometimes the commander or his/her staff lack the time needed to provide the inputs required from a decision maker. The research presented here uses a notional example of stability operations in Iraq during 2007. In the absence of direct inputs from General David Petraeus, the commander of Multi-National Force-Iraq, his September 2007 report to Congress on the situation in Iraq provides the foundation for the case study. After developing an additive value model based on the report, this thesis primarily focuses on quantifying sensitivity analysis for multiple simultaneous weight changes and how they affect the decision.

I.D. Assumptions

Key national and military leaders involved in Iraq are unreachable to solicit their values and objectives for Iraqi stability. Therefore, General David Petraeus's September 2007 report to Congress and subject matter expert (SME) opinion provide the values of the model. Additionally, the value model meets the desirable properties of small size, operability, and is an additive value model.

I.E. Thesis Organization

This thesis consists of five chapters. Chapter 2 reviews literature covering affinity diagramming, value-focused thinking, sensitivity analysis, and the measurement of model robustness. Chapter 3 develops a value model using affinity diagramming and proposes a robust sensitivity analysis technique using five measures. Chapter 4 employs the value model to evaluate and analyze COAs. Finally, the analyst evaluates the COAs' robustness to the weights using the proposed technique. Chapter 5 presents the results of the study, the contributions and limitations of the work, and possible areas of future research.

II. Literature Review

II.A. Introduction

This chapter addresses the past research conducted in the areas of affinity diagramming, value-focused thinking, various analysis techniques, and sensitivity analysis. The use of affinity diagramming, or the similar K-J method, defines the structure of the hierarchy. Value-focused thinking (VFT) develops the value model based on the hierarchy. Sensitivity analysis then evaluates the robustness of the developed value model. Following a brief introduction to joint planning, the chapter examines the details of a few decision analysis studies grouped by steps within the VFT process.

Joint Doctrine 5-0, *Joint Operation Planning*, defines the roles of a Joint Force Commander (JFC) and his/her staff. The JFC serves as the decision maker and ensures the execution of plans for military operations. The role of the JFC's staff consists of supporting the commander in understanding complex systems, planning COAs, recommending COAs to the commander, ensuring mission execution, and mission assessment. This challenge includes "making decisions faster and better than a thinking, adaptive enemy in an environment of uncertainty" (JP 5-0, 2006:Sec. III, 3).

The late Colonel John Boyd, USAF Ret., emphasizes the importance of this concept through his O-O-D-A loop in his series of briefings, "Patterns of Conflict" (Coram, 2002: 334). O-O-D-A stands for Observe, Orient, Decide, Act. Boyd explained that the concept of the O-O-D-A loop applies in any form of competition, whether war, business, sports, and so forth. He further explained that a commander's ability to cycle through the O-O-D-A loop faster than an adversary's O-O-D-A loop enables a

commander to know and counter the adversary's planned actions. This causes confusion and disorientation in the mind of the adversary due to outdated or irrelevant information (Coram, 2002: 335).

The Joint Operation Planning Process (JOPP) consists of seven steps ranging from planning initiation to plan or order development. The seven steps include: Initiation, Mission Analysis, COA Development, COA Analysis and Wargaming, COA Comparison, COA Approval, and Plan or Order Development (JP 5-0, 2006:Sec. III, 20). COA development and comparison fall within the scope of this thesis with a primary focus on a new paradigm for COA comparison.

II.B. Affinity Diagramming

Affinity diagramming is a business tool used to organize thoughts and ideas into a structured hierarchy. The use of affinity diagramming for organization of thought expands beyond the business world. Analysts use the technique to develop decision analysis models based on documents as opposed to input from decision makers or subject matter experts.

Parnell *et al* employed affinity diagramming in the *Air Force 2025 Study*. The team established Gen. Fogleman's statement, "achieve air and space dominance" as the objective of the study (Parnell *et al*, 1998:1340). The inputs received from 40 teams resulted in 109 action verbs (Parnell *et al*, 1998:1340). The team created 14 groups of subtasks and labeled each group with one of the verbs from within the group. Next, the team identified 6 subtasks as tasks and grouped the remaining 8 subtasks into 2 additional tasks (Parnell *et al*, 1998:1341). Finally, the team grouped the 8 tasks under 3 categories

they called functions (Parnell *et al*, 1998:1344). This organization process developed a hierarchical structure suitable for a value-focused thinking approach.

Pruitt models homeland security using a value model structured through affinity diagramming. He extracted 363 objectives from five homeland security documents (Pruitt, 2003:3-15). After grouping common objectives, Pruitt defined the three top objectives as Prevention, Vulnerability Reduction, and Response Preparedness (2003:3-15). Sub-objectives fell accordingly under these top objectives.

Affinity grouping, similar to affinity diagramming, groups common objectives to form a hierarchical structure. Affinity grouping, however, does not follow the grammatical rule of noun-verb pairing. Fensterer uses affinity grouping to develop a value model to evaluate stability operations (2006:53). He uses the 25 tasks found within DoD Directive 3000.05 to establish his hierarchical structure into five main objectives (Fensterer, 2006:56). His sub-objectives come from the 364 tasks identified in three publications from subject matter experts (Fensterer, 2006:56-87,125-142).

II.C. Value-Focused Thinking

Ralph Kenney developed value-focused thinking (VFT) as a proactive decision analysis approach as opposed to a reactive one. Kenney offers a comparison between what he calls alternative-focused thinking (AFT) and VFT (1992:47). AFT, as a reactive decision analysis technique waits for one or more alternatives to arise and then selects the best one. Value-focused thinking allows a decision maker (DM) to be proactive. A DM considers the "things" that are important to him/her. These "things" are values. With the values defined, a DM can generate alternatives that maximize their achievement. When a

decision arises, the DM knows his/her values and is able to generate a list of alternatives that are at least as good as those generated by AFT.

Value-focused thinking implements a ten step process ranging from problem definition to providing recommendations to a DM (Shoviak, 2001:63). The first five steps result in the development of the value model. The next two steps deal with the alternatives. The final three steps are the analysis and recommendations. First, define the objective. The objective provides definition to the problem. Next, develop a value hierarchy. This is where affinity diagramming can come into play to organize the values into a logical hierarchy. Upon completion of the hierarchy, develop evaluation measures for the values in the lowest tiers. The analyst creates value functions to score alternatives with evaluation measures. Often, these functions are single dimensional value functions (SDVFs). Finally, weighting the hierarchy concludes the building process for the value model. The process then turns its attention to the alternatives. Once the value model is complete, generate alternatives to maximize the fulfillment of the DM's objectives. The alternative's fulfillment of each objective are then scored using the value functions. The analysis of the alternatives then begins. Deterministic analysis uses the hierarchy weights and scores from the value functions to find each alternative's overall value score. The alternative with the highest score is preferred. Sensitivity analysis varies one or more weights to determine if the top alternative changes. Finally, the analyst presents the results of the study and provides recommendations to the DM. Military analysts have used this methodology for course of action (COA) authoring and assessment. The remainder of the chapter examines some of those studies.

II.D. VFT for COA Decisions

In 2006, Phipps examined the use of VFT for COA selection from a Department of Defense perspective. She structured her assessment structure based on instruments of national power (Phipps, 2006:29). These instruments include Diplomatic, Information, Military, Economic, and Social (DIMES). She evaluates a diplomatic threat, a conventional option, special operations forces, and a nuclear option as alternatives to achieve a commander's intent in a notional military scenario (Phipps, 2006:34, 36). The approach proved its worthiness based on its adaptability and low strain on time and resources (Phipps, 2006:50-51).

Fensterer applied VFT to the planning and assessment of stability operations in 2007. He developed a value hierarchy through affinity grouping of objectives from doctrine and subject matter experts (Fensterer, 2007:53-91). The value hierarchy developed reflects strategic level objectives, but only incorporates notional evaluation measures (Fensterer, 2007:7, 91-97). He suggests modeling and simulation to improve COA outcome prediction accuracy (Fensterer, 2007:116). The Stabilization & Reconstruction Operations Model (SROM) developed by Robbins provides one possible simulation source. The model assists users in examining the interaction of factors governing the outcomes of stability operations (Robbins, 2005:7-8).

While the theses by Phipps and Fensterer demonstrated the notional use of VFT for COA selection, its use by the military appears frequently in the literature. The applications range from Base Realignment and Closure (BRAC) by the Army and developing future air and space forces by the Air Force to selecting Information Operations (IO) COAs, Psychological Operations (PSYOPS) COAs, and automatic target

recognition systems (Ewing et al, 2006:np; Parnell *et al*, 1998:np; Doyle *et al*, 2000:np; Kerchner *et al*, 2001:np; Bassham *et al*, 2006:np).

The applications examined here all use the same basic methodology, but implement it differently and at different decision levels. Many times, the analysts develop at least an initial value hierarchy structure based on documents and then supplement them with decision maker and subject matter expert opinion. Other applications begin with the decision maker and/or subject matter experts. The weighting technique often varies between projects. The basic VFT process ranks the alternatives, examines the sensitivity of the decision, and then presents the results. The applications examined here, however, often implement additional analysis prior to the sensitivity analysis.

II.E. Value Hierarchy Creation

Value-focused thinking aims to collect the objectives and values of the decision makers to support achieving the objectives, then directly evaluates characteristics of alternatives. Studies often begin when senior decision makers initiate them.

The Air Force 2025 Study began based on statements from then Air Force Chief of Staff, General Ronald R. Fogelman, to the Air University (Parnell *et al*, 1998:1336). The study team initially searched for the gold standard objectives to develop the hierarchy, but implemented the silver standard approach (Parnell *et al*, 1998:1336). Parnell defines gold standard as a model developed from strategic documents and silver standard as models developed based on data from the stakeholder's representatives when gold standard documents are not sufficient and analysts cannot access senior decision makers or stakeholders (2007:626). The team examined gold standard documents such as

the National Security Strategy, National Military Strategy, Defense Planning Guidance, Joint Vision 2010, and others, but none met all the study criteria (Parnell et al, 1998:1339-1340). Instead, the study used affinity diagramming to organize objectives identified by participants (Parnell *et al*, 1998:1340).

In the development of IO COAs, Doyle searched gold standard documents, technical references, and used the opinions of subject matter experts to develop a hierarchy (2000:6). Kerchner, with review by PSYOP experts, developed a value hierarchy to examine the psychological operations based on four US military doctrinal publications (2001:46-47). The 2005 Army BRAC analysis reviewed several government documents relating to defense transformation, stationing, and BRAC for objectives and supplemented them with interviews of stakeholders and key senior military leaders (Ewing et al, 2006:36).

Sometimes, decision makers and stakeholders are accessible. In these instances, analysts directly solicit objectives and the hierarchy structure from the decision maker. Eareckson Air Station in Alaska found itself not meeting regulations for municipal solid waste. Through collaboration with the decision maker, analysts developed a value hierarchy to assist the decision maker in developing environmentally compliant strategies (Chambal *et al*, 2003:25, 27-28). A study to select automatic target recognition (ATR) systems used subject matter experts to develop value hierarchies. A single expert from the Air Force Research Laboratory represented the ATR evaluation community to develop one hierarchy while four subject matter experts from Headquarters Air Combat Command created the warfighter hierarchy (Bassham *et al*, 2006:52).

II.F. Weighting

After completion of the value hierarchy and defining of evaluation measures, the analyst solicits weights for each objective. The weights indicate the decision maker's and/or stakeholder's preferences. The studies examined here used several different weighting techniques.

Air Force 2025 aimed to identify future systems needed in the 2025 timeframe. Analysts did not know the future state of the world; therefore, they developed six possible futures in which to evaluate alternatives (Parnell *et al*, 1998:1346-1347). The authors do not discuss the exact weighting technique used in this application. They do note that each of forty teams submitted weights for the hierarchy for each of the six alternate futures. The weights for the objectives under each future were the average weight across the submissions from all forty teams (Parnell *et al*, 1998:1347). The IO study does not discuss the weighting method used either, but has two representatives each weight the hierarchy for comparative analysis (Doyle *et al*, 2000:9-10).

The tradeoff method involves soliciting the perceived increase in value when a measure moves from its least preferred level to its most preferred level when all other measures are at their least preferred level (Kerchner *et al*, 2001:52). The analyst then rank orders the measures based on value increments and the decision maker allocates percentages to the objectives. Kerchner uses this method to evaluate psychological operations (2001:52-53).

Another weighting technique is swing weighting, or formally known as Simple Multiattribute Rating Technique using Swings (SMARTS) (Ewing *et al*, 2006:41). The ATR system evaluation study uses this method for both the evaluator and warfighter

frameworks (Bassham *et al*, 2006:52, 54). The Army BRAC goes beyond SMARTS to use a Swing Weight Matrix. This technique places all measures into a matrix according to importance and variation. The stakeholders then assign swing weights to all measures. Global weights result from the normalizing of all swing weights in the matrix (Ewing *et al*, 2006: 41-42).

II.G. Additional Analysis

After ranking all alternatives, analysts often conduct additional analysis. Value versus cost reveals dominant alternatives and provides additional insight. The Information Operations study charted the value of each alternative against the associated multiattribute cost value (Doyle *et al*, 2000:11-12). The cost value came from a separate cost hierarchy (Doyle *et al*, 2000:7). Analysts used this same methodology to evaluate psychological operations (Kerchner *et al*, 2001:60-61). In addition to evaluating value versus cost, Doyle *et al* solicited most likely score and range of scores for each attribute (2000:10-11) This allowed him to evaluate COAs while taking the uncertainty of the measures into consideration. Air Force 2025 considered this type of analysis, but future technologies do not have cost associated with them. Instead of cost, Air Force 2025 compares value to the challenge to develop the technology needed for future systems to reach maturity (Parnell *et al*, 1998:1348). Doyle created characteristic plots for the top tier objectives in the hierarchy. These characteristic plots allowed for the comparison of value for each objective between the two decision makers (Doyle *et al*, 2000:12-13). The Army BRAC study wanted to optimize the selection of multiple alternatives subject to a budget constraint, but they did not have the needed information. Instead, the study team

implemented portfolio analysis to determine the minimum mix of alternatives that met the Army's requirements (Ewing *et al*, 2006:42-46).

II.H. Sensitivity Analysis

According to Clemen, sensitivity analysis answers, "What makes a difference in this decision?" (2001:175). Sensitivity analysis can also indicate the robustness of a model under the uncertainty in the weights. A model is most robust to a decision if the ranking of alternatives does not change when weights are varied. The systematic approach to varying the weights is below.

Sometimes, an analyst knows one or more weights with certainty and prefers to keep them constant. Kahraman develops a parametric sensitivity analysis technique using hierarchical value models. The technique holds one or more weights constant while varying the others proportionally using a weight coefficient of elasticity (Kahraman, 2002:32-38). He recommends bounding the coefficients of elasticity to maintain the assumptions of positive weights and the weights summing to one.

An assessment for radioactive disposal reviews five sensitivity techniques before selecting one for implementation (Helton, 1993:327-328). He separates these techniques into one informal and four formal. The informal method described varies one parameter at a time and evaluates the changes that occur. The four formal approaches include differential analysis, Monte Carlo analysis, response surface methodology, and the Fourier amplitude sensitivity test (FAST) (Helton, 1993:327).

Differential analysis uses a Taylor series to approximate the model (Helton, 1993:327). The initial inputs are base values, ranges, and distributions for all variables. A vector of base values forms the starting point for the Taylor series. The Taylor series

approximates the function. Variance propagation techniques use the ranges and distributions to estimate the uncertainty in the approximated function. Finally, the Taylor series estimates the significance of each variable (Helton, 1993:328).

Monte Carlo analysis results from multiple probabilistic model runs. Helton breaks it into five steps (1993:330-331). First, select a range and distribution for each variable. Second, generate a sample from the ranges and distributions for each variable. Third, evaluate the model based on the sample generated in the second step. Fourth, summarize the results for uncertainty analysis, often as a mean and variance. Finally, conduct sensitivity analysis, either through scatterplots or stepwise regression analysis (Helton, 1993:330-331).

Response Surface Methodology (RSM) creates a response surface that serves as a surrogate for the function. Helton breaks RSM into 6 steps (1993:331-332). Again, the process begins by defining a range and distribution for each variable. Second, develop an experimental design to select points for model evaluation. The third step consists of evaluating each point. Fourth, create a response surface based on the results. Fifth, expected value and variance or Monte Carlo analysis estimate the uncertainty in the function. Finally, evaluate the sensitivity of the function to each variable by evaluating the importance of the variables in relation to perturbations of the variables from their expected values (Helton, 1993:331-332).

The FAST approach calculates the expected value and variance of a model prediction through a four step process (Helton, 1993:332). First, define the range and distributions for each variable and use them to construct a density function for the variable. Next, convert the multidimensional integrals for expected value and variance

from the first step into one-dimensional integrals. Then, estimate the expected value and variance using the one-dimensional integrals. Finally, a variable's fractional contribution to variance evaluates the sensitivity of the function (Helton, 1993:332-333).

Bauer *et al* compares the use of response surface methodology (RSM) against tornado diagrams and strategy region graphs for sensitivity analysis of influence diagrams (1999:164). Tornado diagrams, a technique for one-way sensitivity analysis, show a rank order of variability for each factor. Strategy region graphs, a two-dimensional representation of two-way sensitivity analysis, reveal how a decision changes based on the interaction of two factors. He states the benefits of using RSM for sensitivity analysis include a reduced number of sensitivity analysis iterations, an estimation of coefficients at which a decision changes, and insight into the interactions of factors (Bauer *et al*, 1999:162). The study demonstrates the efficiency of RSM and the additional insights gained, compared to other techniques, along with some loss of accuracy resulting from approximations by the RSM equations (Bauer *et al*, 1999:178-179). While not explicitly stated, RSM assumes the role of an n-way sensitivity analysis technique.

Bassham *et al* implements, in an ATR selection study, three sensitivity analysis techniques in conjunction with the value hierarchy (2006:58). A tornado diagram shows a rank order of the variance in values for the evaluation measures. Global weights for each evaluation measure show the importance of each measure, but Bassham notes it provides little insight if multiple changes occur. Finally, he conducts sensitivity analysis using saliency measures. Saliency measures show the contributions of the inputs with

respect to overall value score. While the combat model evaluated is not differentiable, a response surface as a surrogate for the model is (Bassham *et al*, 2006:58-62).

n-way sensitivity analysis is an extension of one- and two-way sensitivity analysis. Hughes demonstrates this technique to assess the stability of a completely subjective decision analysis tree (1990:68). The study analyzes the stability of decisions made by surgical nurses in various situations through case studies. Through the systematic variation of weights, analysts determine a range through which each score can vary. If these ranges do not overlap, then the model demonstrates stability (Hughes, C. and Hughes, K., 1990:68).

Bednarski applied n-way sensitivity analysis to Bayesian networks and characterized it as an NP-hard problem (2003:29). Due to the difficulty of the problem, a genetic algorithm estimates optimal solutions, but cannot guarantee global optima. The study shows the analyzed network insensitive, but investigators hypothesize that this results from the robustness of Bayesian networks (Bednarski et al., 2003: 32).

Rios Insua and French provide a framework for sensitivity analysis in discrete multiobjective decision making. They propose distance-based tools to conduct sensitivity analysis (1991:180). This analysis finds regions in which local optima may lie and eliminates some alternatives due to dominance (Rios Insua and French, 1991:181).

Another approach to sensitivity analysis incorporates mathematical programming techniques. These techniques often evaluate the variation required in initially specified weights to cause the preferred alternative to change. These models look for the new weighting that causes a specific alternative to rank above all others.

Barron and Schmidt first presented a least squares procedure proposing a quadratic programming approach (1988:123-124). The objective function minimizes the sum of the square distance between the initial weight and a new weight. The initial and new weights must sum to one and all weights are positive (Barron and Schmidt, 1988:123). A smaller objective function value indicates greater sensitivity.

Wolters and Mareschal propose a similar approach using goal programming. Each weight has two deviational variables indicating an increase or decrease from the initial weight. The objective function tries to minimize the sum of these deviational variables (Wolters and Mareschal, 1995:284). They also show constraints used to preserve relative importance of weights, penalize deviational variables with cost coefficients, and place upper and lower bounds on weights (Wolters and Mareschal, 1995:284-285).

Ringuest extends the work of Barron and Schmidt and Wolters and Mareschal to a generalized L_p metric. Ringuest titles the approach of Barron & Schmidt the L_1 metric (1997:566-567). Ringuest strengthens this metric by combining it with an L_∞ metric. The L_∞ metric minimizes the maximum deviation of a single weight. The deviations of all other weights remain less than this maximum deviation (Ringuest, 1997:567). Using compromise programming, Ringuest minimizes both his L_1 and L_∞ metric and expresses it as the L_p metric (Ringuest, 1997:566).

This thesis focuses on the examination of measures defined by Barron and Schmidt, Wolters and Moreschal, and Ringuest's L_1 and L_∞ metrics.

II.I. Summary

This chapter reviews the literature in the fields of affinity diagramming, value-focused thinking, and sensitivity analysis. Repeatedly, affinity diagramming proves worthy for organizing value-hierarchies in the absence of an overarching decision maker. While VFT is a complete methodology for analysis; analysts often link the method with other tools like portfolio analysis. A key point to each study using VFT is the sensitivity analysis. Analysts conduct sensitivity analysis using informal, ad hoc, approaches and more formal approaches ranging from single factor to multi-factor analysis. The next chapter builds the methodology of this study, demonstrating the use of VFT to author and assess course of action and a robust sensitivity analysis approach.

III. Methodology

III.A. Introduction

This chapter addresses the methods used to develop a subjective value hierarchy and evaluate its robustness. Initially, VFT structures a commander's objectives for course of action selection and assessment into a hierarchy. The technique of affinity diagramming provides a starting point in the absence of a specific decision maker. This research then constructs a value model based on the developed hierarchy. The value model provides guidance to develop COAs. Finally, this research develops sensitivity measures for all alternatives to indicate how the COA is sensitive to small changes in the weights.

III.B. Extracting Needed Information

Decision analysis typically develops decision models based on a decision maker's inputs. This is not always the case as seen in the *Air Force 2025 Study* (Parnell *et al*, 1998:1340). When analysts cannot reach senior decision makers due to location or time constraints, they can base models on the available information, sometimes in the form of policy documents or speeches by decision makers. These sources of information often contain the objectives of the decision maker, measures that quantify the achievement of objectives, and possible actions to achieve the stated objectives. When extracting this information, separating it into these three categories provides organization for building the model. Capturing this information on index cards or similarly sized pieces of paper assists in the later organization process (Alloway, 1997:75-76).

Stated goals and objectives in the source should reveal what a decision maker's values are in a situation. This information enables the analyst to begin building a value

model. Writing down each goal or objective from the source on a separate index card and including the source from which it came provides traceability of the process. Each card should follow the same grammatical format, such as a verb followed by at least a noun. This format lends itself to the affinity diagramming procedure detailed in the next section.

Measures indicate an achievement of goals or objectives. Measures included in the documents or speeches provide a starting point by showing what is currently used. In addition to capturing what is measured, it is useful to capture current data associated with those measures as well as the desired level of the measure.

The final piece of information to capture is possible actions. These actions provide a starting point to develop alternatives for the decision model. An alternative can also consist of multiple actions. After capturing all the needed information, affinity diagramming organizes the goals and objectives.

This research uses General Petraeus's September 2007 *Report to Congress on the Situation in Iraq* which addresses the United States military's stability objectives in Iraq, possible actions, and measures used to gauge success. Extracting the objectives in a verb-noun phrase supports the affinity diagramming process, the next step in building the decision model. The overall objective described by the report is a stable Iraq (Petraeus, 2007:2). Table 3.1 shows the additional information extracted from the report.

Table 3.1 Extracted Information from Petraeus Report

Objectives:	
- Reduce ethno-sectarian violence/deaths - Reduce overall civilian deaths - Reduce number of insurgent crossing border	-Grow Iraqi Security Forces & shoulder the load -Establish joint security stations
Actions:	
- Take away insurgent sanctuaries and gain the initiative - Disrupt Shia militia extremists -- capturing the head and other leaders of Iranian supported special groups -- neutralize 5 media centers -- detained senior leaders -- captured ~100 other key leaders & 2500 rank-and-file fighters - Dialog with insurgent groups and tribes	- Percent Coalition troops support training of Indigenous Security Institution (Civil Defense, Indigenous military, Border Patrol, Facility Protection) -- Number of training classes -- Number of facilities - Humanitarian Relief - Support to critical infrastructure
Measures:	
- Tribal rejection of Al-Qaeda - No. of overall civilian deaths - No. of ethno-sectarian deaths - No. of attacks (car bombings, suicide) - Coalition losses	- Willingness of locals to serve in the Army and Police Service -- 140 Army, Police and Special Operations Forces -- 95 capable of taking the lead - 445,000 individuals on payrolls for Iraq's government

While the information extracted does not provide the entire model, it does offer a starting point.

III.C. Affinity Diagramming

Affinity diagramming is a business tool used to organize thoughts and ideas into a structured hierarchy. Based on a sentence that summarizes the overall problem, participants gather supporting topics from documentation with descriptions consisting of a noun-verb pair at a minimum (Brassard and Ritter, 1994:13). After exhaustively gathering support topics, grouping similar topics together, and writing a sentence to

summarize the topics within the group begins the organization process. Grouping similar summarized topics and again summarizing these new groups begins developing the hierarchical structure needed for the value model. Figure 3.1 provides an illustration of the hierarchical structure. Alloway suggests using index cards for the process for easy reorganization at any point (1997:75-76).

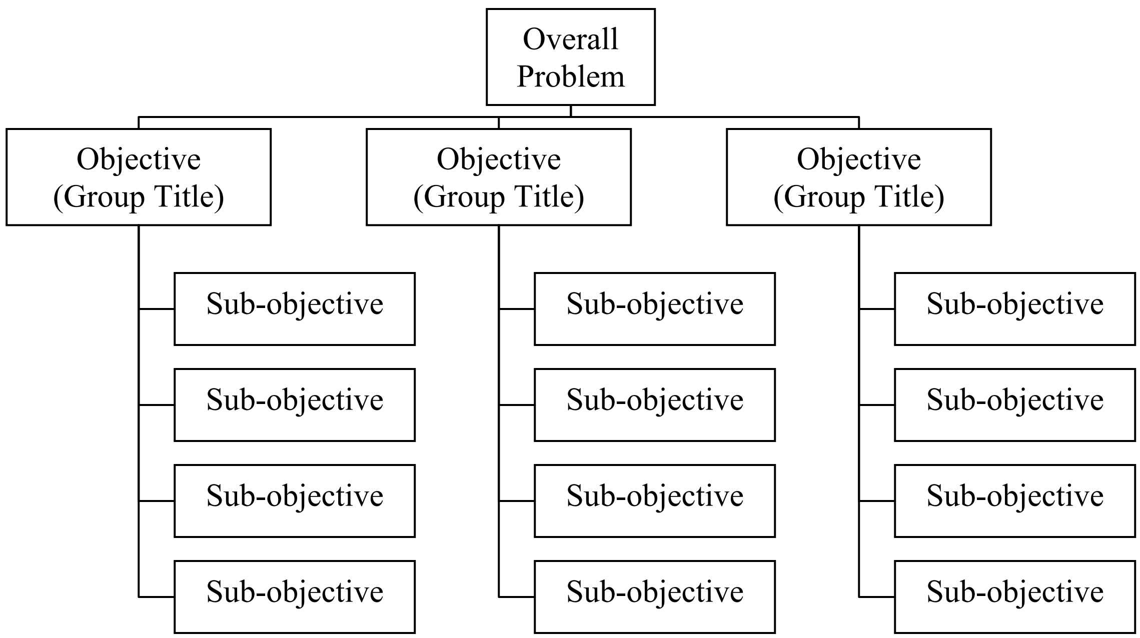


Figure 3.1 Hierarchical Structure from Affinity Diagramming

Brassard refers to the K-J method as a variation on affinity diagramming. The key differences include the organization of fact-based ideas and a more structured process in the K-J method (Brassard and Ritter, 1994:16). The K-J method, invented in the early 1950's in Japan by Jiro Kawakita, achieves the same goal as affinity diagramming (Scupin, 1997:234). Kawakita invented the method when trying to synthesize large amounts of data. Like affinity diagramming, the user writes topics on index cards, but then shuffles them. Shuffling index cards reduces bias. The next step is grouping the

cards by topics, similar to affinity diagramming, and summarizing the groups. Finally, drawing the hierarchy based on the groupings provides the basic structure for the value model (Scupin, 1997:236).

Examining the objectives from Table 3.1 indicates a relationship between the first three objectives and another relationship between the last two. The first three objectives seem related to security while the last two relate to supporting transition within Iraq. Based on the information thus far, the hierarchy begins taking form as seen in Figure 3.2.

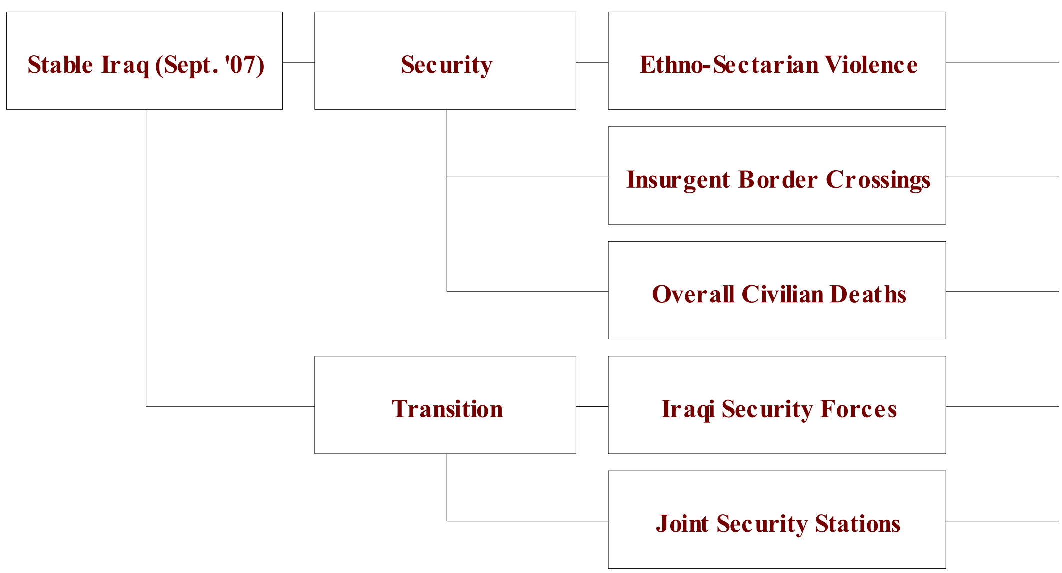


Figure 3.2 Affinity Diagram based on Petraeus Report

The affinity diagram provides the initial framework for value hierarchy developed in the next section.

III.D. Value Hierarchy

Affinity diagramming may provide initial organization and structure for a value hierarchy, but the initial hierarchy may lack certain desirable properties. The value

hierarchy includes the objectives from the affinity diagram, refines the structure, and adds the value measures. Kirkwood lists the five desirable properties of a value hierarchy in his book *Strategic Decision Making* (1997:16-19). These properties include: Completeness, Nonredundancy, Decomposability, Operability, and Small size.

Completeness, also termed collectively exhaustive, ensures that each level of the hierarchy includes all the information needed to evaluate the next higher level (Kirkwood, 1997:16). Nonredundancy, or mutual exclusivity, ensures that evaluation considerations do not overlap (Kirkwood, 1997:16-17). Decomposability, or preferential independence, ensures that a change in one evaluation measure does not cause a change in another evaluation measure (Kirkwood, 1997:17-18). Preferential independence allows for the use of an additive value function for the value model. Operability ensures that the value hierarchy is understandable and usable in the eyes of the decision maker (Kirkwood, 1997:18). The final desired property, small size, emphasizes the point that it is easier to explain and understand a smaller hierarchy as opposed to a larger one (Kirkwood, 1997:18).

The evaluation measures show the level of achievement for the lowest level objectives. Each measure has a combination of two classifiers. Measures have either a natural or constructed scale, and they have either a direct or proxy scale (Kirkwood, 1997:24-25). The four combinations of these classifiers rank from most preferred, natural-direct, to least preferred, constructed-proxy. Natural measures are generally interpreted the same by everyone, such as profit in dollars. Analysts develop constructed scales for a specific decision problem and explicitly define the meaning of the scale. Direct scales directly measure the level of achievement for an objective, again such as

profit in dollars, while proxy scales show a level of achievement for an objective, but do not directly measure it, such as gross domestic product of a country (Kirkwood, 1997:24).

The first desirable property of a value hierarchy is completeness. Forming objectives based on the actions and measures extracted from the report allows for a more complete hierarchy. Adding objectives such as critical infrastructure and humanitarian relief makes the hierarchy more complete. The next desirable property is nonreduncancy. Overall civilian deaths depends on the deaths associated with ethno-sectarian violence, and therefore violates the nonredundancy property. Separating civilian deaths into those caused by ethno-sectarian and non-sectarian violence satisfies this property. The sub-objectives under *Unit Capability Levels* in the hierarchy below present problems relating to decomposability. As a unit becomes more or less proficient in their training, they move from one capability level to the other. A change in the measure for one capability level has an associated change in the measure of another capability level. This is an example of preferential dependence in the model. Simulation, however, shows that consistent and reliable results are achievable as long as the value model is close to being preferentially independent (Stewart, 1996:308). Operability and small size are subjective properties with their satisfaction determined by the decision maker. The assumption is that the value model established in this research is operable and small.

After modifying the hierarchy to meet all objectives and linking measures to each objective, the hierarchy takes the form shown in Figure 3.3.

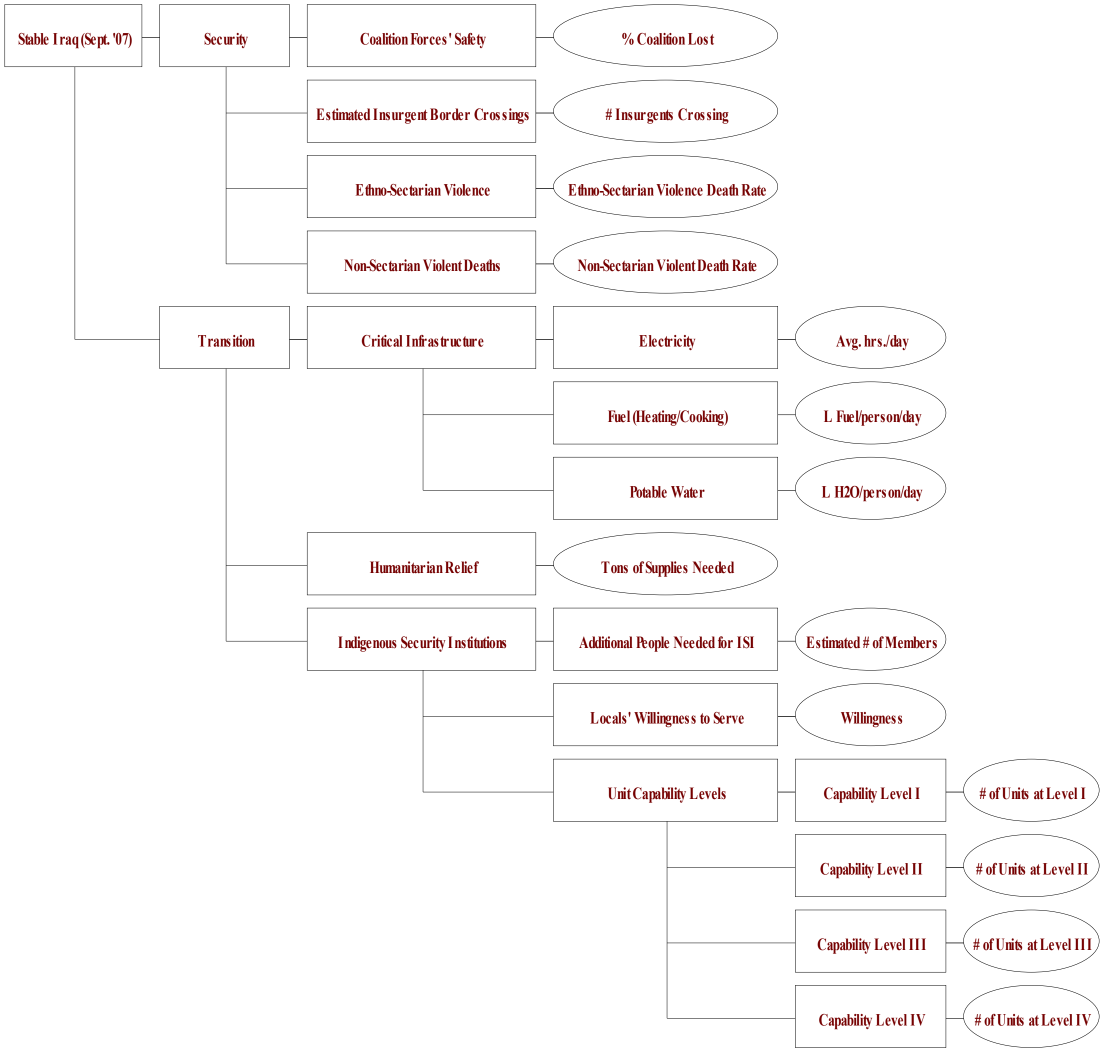


Figure 3.3 Value Hierarchy based on Petraeus Report

Appendix A contains information pertaining to the evaluation measures including their scales. This model uses fourteen attributes to measure a course of action's effectiveness in achieving the overall objective, a stable Iraq.

III.E. Single Dimensional Value Functions

Single dimensional value functions (SDVFs) translate evaluation measures to value scores ranging between zero and one. They may be continuous or discrete.

Continuous forms include linear, piecewise linear, exponential, or S curve. Discrete SDVFs are broken into bins. Whether SDVFs are increasing or decreasing, they must be monotonic. Graphs of these value functions place the independent evaluation measure range on the x-axis with the value ranging between zero and one on the y-axis.

Linear value functions provide proportional translations from evaluation measure to value. Increasing linear functions follow equation 3.1a while decreasing linear functions follow equation 3.1b shown below:

$$v(x_i) = \frac{x_i - Low}{High - Low} \qquad \text{Equation 3.1a}$$

$$v(x_i) = \frac{High - x_i}{High - Low} \qquad \text{Equation 3.1b}$$

Inserting the evaluation measure for x_i provides the value score for the evaluation measure (Kirkwood, 1997:65-66).

Discrete functions divide an evaluation measure into bins. Categorical measures lend themselves to this type of function. The least preferred category receives a value of zero while the most preferred receives a value of one. Assign the value to the remaining categories using value increments (Kirkwood, 1997: 62). For example, if there are 3 categorical measures, A, B, and C, where A is the least preferred and C is the most preferred, then A has a value of zero and C has a value of one. B is more important than A by a value increment of x. C is more important than B, but by twice the increment between A and B. The increment between A and B is x and the increment between B and C is $2x$, then these increments summed together are $3x$ over a range of 1. Solving for x, 1 divided by 3 results in x equaling 0.33, meaning B has a value of 0.33.

Piecewise linear functions allow continuous translation over categorical measures with different rates for each range. The SDVF has a different linear equation for each range of the evaluation measure and corresponding value. The method of value increments determines the values at which the slope of the piecewise linear function changes. Increasing functions follow equation 3.2a for all ranges while decreasing functions follow equation 3.2b shown below:

$$v(x_i) = v_{RangeLow} + (v_{RangeHigh} - v_{RangeLow})\frac{x_i - Low}{High - Low} \qquad \text{Equation 3.2a}$$

$$v(x_i) = v_{RangeLow} + (v_{RangeHigh} - v_{RangeLow})\frac{High - x_i}{High - Low} \qquad \text{Equation 3.2b}$$

High/Low and RangeHigh/RangeLow correspond with the high and low evaluation measure and value for each piecewise linear section of the overall SDVF.

Exponential functions can estimate a piecewise linear function over the range of the evaluation measure and simplify the solicitation process since it requires only three data points. One adjusts the curvature of the function by changing ρ in equation 3.3a for increasing exponential functions or 3.3b for decreasing functions.

$$v(x_i) = \frac{1 - e^{\frac{-(x_i - Low)}{\rho}}}{1 - e^{\frac{-(High - Low)}{\rho}}} \qquad \text{Equation 3.3a}$$

$$v(x_i) = \frac{1 - e^{\frac{-(High - x_i)}{\rho}}}{1 - e^{\frac{-(High - Low)}{\rho}}} \qquad \text{Equation 3.3b}$$

Positive values of ρ cause the function to be concave while negative values of ρ produce a convex function. As ρ approaches infinity, the function approaches linearity. Increasing functions with a negative ρ gives more value to evaluation measures higher in

the range; positive ρ gives more value to lower evaluation measures. Decreasing functions work in the opposite manner with negative ρ giving more value to evaluation measures closer to the low end of the range (Kirkwood, 1997:65-66). The S-curve finds its basis in the exponential function, but changes convexity at a point within the range of the evaluation measure resulting in half opening concave up and the other half concave down. Piecewise linear, exponential, and S-curve SDVFs allow for varying returns to scale from the measure to the associated value.

This section details the development of the *Estimated Insurgent Border Crossing* evaluation measure single dimensional value functions (SDVF). Appendix B contains the remainder of the SDVFs. Since the focus of this research is the approach to robust sensitivity analysis, the author does not detail them here. The evaluation measures range from zero, the most preferred and mapped to a value of 1, to one hundred, the least preferred and mapped to a value of 0. The subject matter expert (SME) identified that 10 insurgents crossing the border per month achieves a value of 0.5. He felt that 10 or more insurgents entering in a month could result in the formation of a new terrorist cell. Given the three points and based on Equation 3.3b, fitting a decreasing exponential curve to the data points results in a ρ of –14.4475, shown in Figure 3.4.

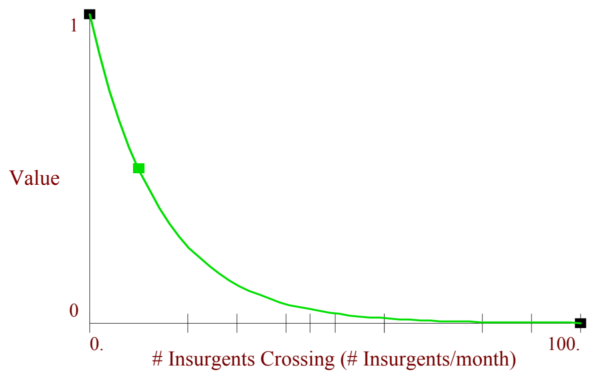


Figure 3.4 Estimated Insurgent Border Crossings SDVF

III.F. Weighting the Hierarchy

Assigning weights to attributes in the hierarchy recognizes relative importance between the attributes. The sensitivity analysis techniques described in this research apply to global weights solicited locally using swing weights. Local weights quantify a decision maker's preference between objectives within a single tier of the hiearachy. Swing weights recognize the ranges of attributes (von Winterfeldt and Edwards, 1986:275). Solicitation of the weights involves asking a decision maker to first rank, from least important to most important, the evaluation measure range for all attributes in a single tier of the hierarchy. The least important attribute receives the value of *k*. Progressing in ascending order of the attributes, the decision maker offers how much more important it is to swing the attribute from its lowest measure to its highest measure as opposed to doing the same with the least important attribute. The attribute receives the value of y*k*, where y indicates the relative importance as a multiple of changes in the least important attribute. After soliciting the relative importance for all attributes in a tier of the hierarchy, divide 1 by the sum of the y*k*'s. This provides the local weight for the least

important attribute in the tier. The remaining attribute weights result from multiplying the corresponding *y* by the value of *k* as seen in equation 3.4.

$$W_i = y_i k \frac{1}{\sum_{i=1}^{n} y_i k} \qquad \text{Equation 3.4}$$

After calculating the local weights in all tiers of the hierarchy, one can then calculate the global weights. The global weights are the product of the local weights starting with an attribute at the lowest tier and multiplying by the local weight from the tier above it until reaching the top of the hierarchy. After calculating all global weights, they sum to one.

Starting with the first tier of the hierarchy, *Security* and *Transition*, the SME asserts that *Transition* is least important and assigned a 1*k* while *Security* is 3.5 times as important as *Transition* and assigned a 3.5*k*. The two sum to 4.5 with a reciprocal of 0.222. This results in local weights of 0.222 for *Transition* and 0.778 for *Security*. Table 3.2 shows the relative importance within each tier of the hierarchy along with the associated local weights.

Table 3.2 Relative Importance and Local Weights

Objective	**Relative Importance**	**Local Weight**
Security	3.5	0.778
Transition	1	0.222

Objective	**Relative Importance**	**Local Weight**
Coalition Forces' Safety	2	0.276
Estimated Insurgent Border Crossings	1	0.138
Ethno-Sectarian Violence	2.5	0.345
Non-Sectarian Violent Deaths	1.75	0.241

Objective	Relative Importance	Local Weight
Critical Infrastructure	1	0.103
Humanitarian Relief	6.25	0.641
Indigenous Security Institutions	2.5	0.256

Objective	Relative Importance	Local Weight
Electricity	2	0.333
Fuel (Heating/Cooking)	1	0.167
Potable Water	3	0.500

Objective	Relative Importance	Local Weight
Additional People Needed for ISI	1	0.143
Locals' Willingness to Serve	2	0.286
Unit Capability Levels	4	0.571

Objective	Relative Importance	Local Weight
Capability Level I	6	0.500
Capability Level II	3	0.250
Capability Level III	2	0.167
Capability Level IV	1	0.083

The sensitivity analysis methods used in this research require global weights. An attribute's global weight is the product of its local weight and the attributes beneath which it falls. For example, *Estimated Insurgent Border Crossings* has a local weight of 0.138 and *Security* has a local weight of 0.778, so *Estimated Insurgent Border Crossings* has a global weight of 0.107. Table 3.3 shows the global weights in rank order for the fourteen attributes that have an evaluation measure attached to them. Figure 3.5 shows the global weights on the value hierarchy.

Table 3.3 Global Weights

Objective	Global Weight
Ethno-Sectarian Violence	0.268
Coalition Forces' Safety	0.215
Non-Sectarian Violent Deaths	0.188
Humanitarian Relief	0.142
Estimated Insurgent Border Crossings	0.107
Locals' Willingness to Serve	0.016
Capability Level I	0.016
Potable Water	0.011
Additional People Needed for ISI	0.008
Capability Level II	0.008
Electricity	0.008
Capability Level III	0.005
Fuel (Heating/Cooking)	0.004
Capability Level IV	0.003

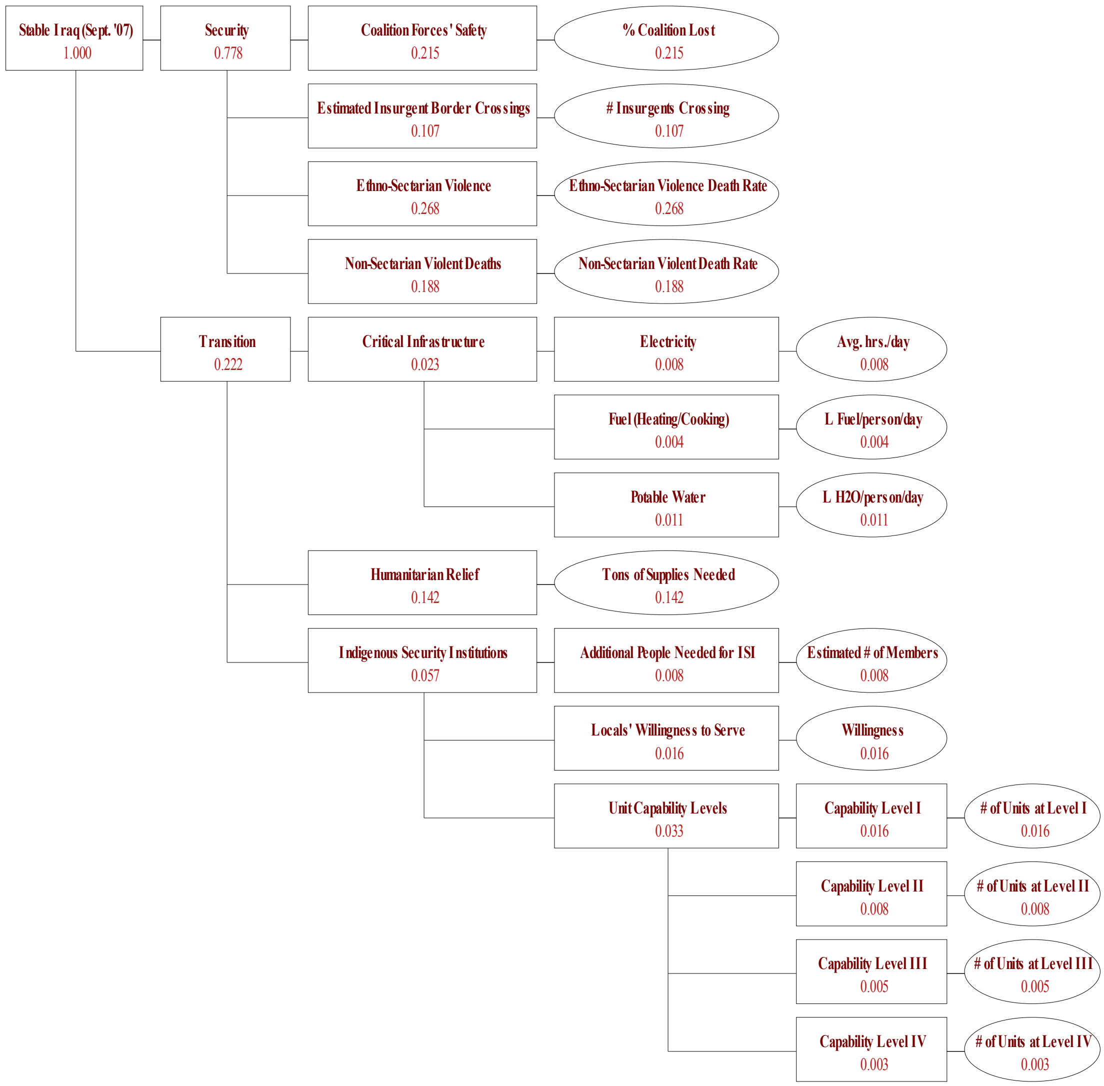


Figure 3.5 Value Hierarchy with Global Weights

III.G. Course of Action Development

Courses of action describe the proposed approaches to achieving the decision maker's overall objective. Each course of action has a score for all evaluation measures. The evaluation scores translate into values using the corresponding SDVF. One can then calculate the value score for the alternatives. Based on the additive value function seen in equation 3.5, the value for an alternative is the sum of the product of the value score for

each attribute according to its SDVF and its corresponding weight. The superscript on V differentiates the different alternatives.

$$V^{I} = \sum_{i=1}^{k} W_{i} v\left(x_{i}\right) \qquad \text{Equation 3.5}$$

The courses of action are then ready for ranking to discover the preferred order of alternatives.

The notional example in this research evaluates ten alternatives. The *Status Quo* course of action characterizes the situation in Iraq in September 2007. The other nine alternative courses of action propose different approaches that may improve on the achievement of the overall objective compared to the status quo. The ten proposed courses of action, in no particular order include:

- Status Quo
- Expel Al Qaeda-Iraq (AQI)
- Train Indigenous Security Institutions (ISI)
- Institute a Military Draft in Iraq
- Partial Coalition Withdrawal from Iraq
- Full Coalition Withdrawal from Iraq
- Pursue Kurdistan Workers Party (PKK) terrorists in Northern Iraq
- Establish Self-Sustained Agriculture within Iraq
- Establish 24 hour Electricity throughout the country
- Lower Iraq's Unemployment Rate

Each course of action has an evaluation score for each of the fourteen attributes shown in Table 3.3. The evaluation scores characterize each course of action in order to

differentiate between them. They enable the value hierarchy to evaluate the courses of action. Table 3.4 shows the notional evaluation measures for the alternatives.

Table 3.4 COA Evaluation Measures

	% Coalition Lost	# Insurgents Crossing	Ethno-Sectarian Violence Death Rate	Non-Sectarian Violent Death Rate	Avg. hrs./day
Status Quo	2	50	40	35	13
Expel AQI	5	20	20	30	13
Train ISI	1	30	20	30	15
Institute Draft	4	60	50	50	11
Partial Coalition Withdrawal	1	90	40	40	12
Full Coalition Withdrawal	0	100	50	60	8
Pursue PKK	10	80	50	70	14
Self-Sustained Agriculture	2	60	25	35	13
24 hr Electricity Established	2	40	20	35	24
Lower Unemployment	2	45	25	35	13

	L H2O/person /day	L Fuel/person/ day	Tons of Supplies Needed	Estimated # of Members	Willingness
Status Quo	47	0.32	270	40000	High
Expel AQI	60	0.50	270	20000	High
Train ISI	65	0.45	270	20000	High
Institute Draft	45	0.20	270	10000	Low
Partial Coalition Withdrawal	46	0.30	270	60000	Medium
Full Coalition Withdrawal	45	0.15	270	90000	Low
Pursue PKK	55	0.25	270	50000	Medium
Self-Sustained Agriculture	50	0.37	50	35000	High
24 hr Electricity Established	80	0.70	150	30000	High
Lower Unemployment	70	0.40	100	25000	Medium

	# of Units at Level I	# of Units at Level II	# of Units at Level III	# of Units at Level IV
Status Quo	12	83	42	23
Expel AQI	15	85	40	20
Train ISI	50	90	15	5
Institute Draft	12	83	50	30
Partial Coalition Withdrawal	12	43	21	67
Full Coalition Withdrawal	12	10	10	118
Pursue PKK	15	85	40	20
Self-Sustained Agriculture	12	83	42	23
24 hr Electricity Established	12	83	42	23
Lower Unemployment	12	83	42	23

III.H. COA Scoring and Ranking

After calculating the value score for each alternative, ranking them in descending order provides the preferred order of the alternatives. The alternative with the highest value is the preferred alternative. At this point, sensitivity analysis examines the alternatives and how changes in global weights may change the preferred alternative. This analysis shows robustness of the preferred alternative in the face of uncertainty in the weights.

After assessing the evaluation scores for all alternative courses of action, quantitative comparison between them can begin. SDVFs translate each evaluation score to an associated value ranging between 0 and 1. A course of action's value score is the sum product of the fourteen global weights in Table 4.2 and associated values derived by the SDVFs. The value scores also range between 0 and 1. An optimal alternative has a value score of 1 because it scores a 1 on every attribute. A rank order of courses of action based on value score in descending order shows the preferred order of alternatives.

III.I. Traditional Sensitivity Analysis

One-way sensitivity analysis selects one factor at a time and varies the weight from zero to one. All other weights change proportionally to maintain the relative weighting between all factors. Equation 3.6 shows the equation for finding the new weights w_i while varying W_k between zero and one, where k indicates the weight being varied (Kirkwood, 1997:82, 84).

$$w_i = \frac{W_i}{\sum_{\substack{i=1 \\ i \neq k}}^{n} W_i} \qquad \text{Equation 3.6}$$

One-way sensitivity analysis graphically shows a change in decision when the value lines of an alternative cross in two dimensional space. The top line in the graph shows the preferred decision over the specified weight range. Kirkwood explains the process involved in applying one-way sensitivity analysis to a value hierarchy (1997:82, 84-85). This approach requires a separate chart for each weight in the model which can become cumbersome for decision problems with many factors.

Two-way sensitivity analysis selects two factors and varies both of them from zero to one. This approach examines the interaction between the two selected factors. All weights must still sum to one. Graphically, two-way sensitivity is three dimensional. It is clear that the graphical representation of sensitivity analysis is one dimension greater than the number of weights varied. This limits graphical representation of sensitivity analysis to only two factors.

III.J. Robust Sensitivity Measures

While one-way sensitivity analysis can indicate the sensitivity of alternatives to a particular weight, it does not take multiple non-proportional weight changes into consideration. Two-way sensitivity analysis can change two weights in a non-proportional manner while the remaining weights change proportionally such that they all continue to sum to one, but this requires the problem to have only three factors. Since all weights can vary between zero and one with the only constraint that they must sum to one, the possibilities of weight combinations are infinite.

Small changes in global weights indicate high decision sensitivity. Mathematical programming can minimize a defined measure measuring the changes across all weights while meeting defined constraints. The research here takes this approach while using

four pre-defined measures and presents a fifth. Each mathematical program minimizes the measure, while ensuring the value score for an alternative ranks it above all other alternatives, the new set of weights sums to one, and all weights are non-negative. This requires solving a mathematical program for each alternative for each measure. If a problem is infeasible, then the alternative cannot rank first and is not sensitive.

Barron and Schmidt first presented a least squares measure for sensitivity analysis of multiattribute value models (1988:123). The measure is the sum of the squared distance between the original weights, W_i from Table 3.3, and the new weights, w_i, that minimize this measure. The quadratic program formulation follows:

Problem 3.1a Minimize $\sum_{i=1}^{k} (W_i - w_i)^2$

subject to :

$$\sum_{i=1}^{k} (v^A(x_i) - v^B(x_i))\, w_i \geq 0 \qquad \forall A \neq B$$

$$\sum_{i=1}^{k} W_i = 1$$

$$\sum_{i=1}^{k} w_i = 1$$

$$0 \leq w_i, W_i \leq 1 \qquad \forall i = 1..k$$

where:

W_i = the initial weights defined by the decision maker
w_i = the weights found that minimize the measure
$v^A(x_i)$ = value score of attribute i for alternative A

The problem has 2k + n + 1 constraints, where k is the number of attributes and n is the number of alternatives. Problem 3.1a has n – 1 constraints that force an alternative to rank above all others. There are constraints requiring the initial and new weight scheme to individually sum to one. Finally, there are 2k boundary constraints requiring the initial and new weights to range between zero and one.

Wolters and Mareschal asserted a linear programming approach to sensitivity analysis of additive multiple criteria decision making methods (1995:284). The measure is the sum of the deviational distances between the original weights and the new weights that minimize this measure. This is the 1-norm of the vector of changes in weights. The linear program formulation follows:

Problem 3.1b

$$\text{Minimize } \sum_{i=1}^{k}(s_i^+ + s_i^-)$$

subject to :

$$\sum_{i=1}^{k}(v^A(x_i) - v^B(x_i))(W_i + s_i^+ - s_i^-) \geq 0 \quad \forall A \neq B$$

$$\sum_{i=1}^{k} W_i = 1$$

$$\sum_{i=1}^{k}(s_i^+ - s_i^-) = 0$$

$$s_i^+, s_i^- \geq 0 \quad \forall i = 1..k$$

$$0 \leq W_i + s_i^+ - s_i^- \leq 1 \quad \forall i = 1..k$$

where:

W_i = the initial weights defined by the decision maker
s_i^+ = the positive deviational distance from W_i
s_i^- = the negative deviational distance from W_i
$v^A(x_i)$ = value score of attribute i for alternative A

This problem has 3k + n + 1 constraints, where k is the number of attributes and n is the number of alternatives. There are n – 1 constraints requiring an alternative to rank above all others by changing one deviational variable or the other for each weight. The initial weights must sum to one and the sum of the differences between deviational weights for each attribute must sum to zero. There are 2k constraints requiring the non-negativity of deviational variables and an additional k constraints bounding the weights between zero and one.

Ringuest extended Wolters and Mareschal's linear programming approach by adding the infinity norm (1997:566-567). The measure minimizes the maximum positive or negative deviation from initial weights. The linear program formulation follows:

Problem 3.1c

$$
\begin{aligned}
&\text{Minimize } D \\
&\text{subject to:} \\
&\sum_{i=1}^{k}(v^{A}(x_i) - v^{B}(x_i))(W_i + s_i^{+} - s_i^{-}) \geq 0 \quad \forall A \neq B \\
&s_i^{+} + s_i^{-} \leq D \quad \forall i = 1..k \\
&\sum_{i=1}^{k} W_i = 1 \\
&\sum_{i=1}^{k}(s_i^{+} - s_i^{-}) = 0 \\
&s_i^{+}, s_i^{-} \geq 0 \quad \forall i = 1..k \\
&0 \leq W_i + s_i^{+} - s_i^{-} \leq 1 \quad \forall i = 1..k
\end{aligned}
$$

where:

D = the maximum deviation from an initial weight W_i
W_i = the initial weights defined by the decision maker
s_i^{+} = the positive deviational distance from W_i
s_i^{-} = the negative deviational distance from W_i
$v^A(x_i)$ = value score of attribute i for alternative A

Problem 3.1c has 4k + n + 1 constraints, where k is the number of attributes and n is the number of alternatives. There are n – 1 constraints requiring an alternative to rank above all others by changing one deviational variable or the other for each weight. k constraints ensure that the maximum deviation of any weight is at least as small as the objective function value. The initial weights must sum to one and the sum of the differences between deviational weights for each attribute must sum to zero. There are 2k constraints requiring the non-negativity of deviational variables and an additional k constraints bounding the weights between zero and one.

Bauer proposed the use of the 2-norm as a measure for sensitivity analysis (2008:np). Similar to Baron and Schmidt's least squares method, the 2-norm minimizes

the square root of the sum of the squared distance between the original weights and the new weights that minimize this measure. The quadratic program formulation follows:

Problem 3.1d Minimize $\sqrt{\sum_{i=1}^{k}(W_i - w_i)^2}$

subject to :

$$\sum_{i=1}^{k}(v^A(x_i) - v^B(x_i))\, w_i \geq 0 \quad \forall A \neq B$$

$$\sum_{i=1}^{k} W_i = 1$$

$$\sum_{i=1}^{k} w_i = 1$$

$$0 \leq w_i \leq 1 \quad \forall i = 1..k$$

where:

W_i = the initial weights defined by the decision maker
w_i = the weights found that minimize the measure
$v^A(x_i)$ = value score of attribute i for alternative A

This problem, like the least squares problem, has 2k + n + 1 constraints, where k is the number of attributes and n is the number of alternatives. Problem 3.1d has n – 1 constraints that force an alternative to rank above all others. There are constraints requiring the initial and new weight scheme to individually sum to one. Finally, there are 2k boundary constraints requiring the initial and new weights to range between zero and one.

The infinity norm demonstrated by Wolters and Mareschal captures one important piece concerning a change in weight in terms of sensitivity. When global weights are small, such as 0.01, a change to 0.02 is a change of 0.01. This is however, a 200% change in weight. The new proposed approach minimizes the maximum percent change across all weights. The linear program formulation follows:

Problem 3.1e

$$\text{Minimize } P$$
$$\text{subject to:}$$
$$\sum_{i=1}^{k}(v^A(x_i) - v^B(x_i))(W_i + s_i^+ - s_i^-) \geq 0 \quad \forall A \neq B$$
$$\frac{(s_i^+ + s_i^-)}{W_i} \leq P \quad \forall i = 1..k$$
$$\sum_{i=1}^{k} W_i = 1$$
$$\sum_{i=1}^{k}(s_i^+ - s_i^-) = 0$$
$$s_i^+, s_i^- \geq 0 \quad \forall i = 1..k$$
$$0 \leq W_i + s_i^+ - s_i^- \leq 1 \quad \forall i = 1..k$$

where:

P = the maximum percent change of an initial weight W_i
W_i = the initial weights defined by the decision maker
s_i^+ = the positive deviational distance from W_i
s_i^- = the negative deviational distance from W_i
$v^A(x_i)$ = value score of attribute i for alternative A

Problem 3.1e has 4k + n + 1 constraints, where k is the number of attributes and n is the number of alternatives. There are n – 1 constraints requiring an alternative to rank above all others by changing one deviational variable or the other for each weight. The k constraints ensure that the maximum percent change of any weight is at least as small as the objective function value. The initial weights must sum to one and the sum of the differences between deviational weights for each attribute must sum to zero. There are 2k constraints requiring the non-negativity of deviational variables and an additional k constraints bounding the weights between zero and one.

This research uses Frontline System's Premium Solver for Education V6.0 add-in with Excel 2003. The software has three engines for solving mathematical programs. This research used the *Standard Simplex LP* engine for solving the linear program associated with the Problem 3.1b. Solving the quadratic programs for Problems 3.1a and 3.1d required the *Standard GRG Nonlinear* engine. This engine uses a generalized reduced gradient (GRG) method to solve the quadratic programs. Finally, the

mathematical programs with non-smooth objective functions, Problems 3.1c and 3.1e, required the use of the *Standard Evolutionary* engine. This third engine is a heuristic approach that estimates the optimal solution to the mathematical program and it cannot guarantee its optimality. The heuristic approach is a combination of both genetic and evolutionary algorithms along with classical optimization methods (Premium Solver V8, 2007:26). While this section introduced the measures used and their calculations, the next section details the sensitivity analysis using these measures.

III.K. Robust Sensitivity Analysis

The five measures detailed in the previous section provide a way to measure the sensitivity of alternatives to small changes in multiple weights. Smaller measure values indicate greater sensitivity. The top alternative according to the value model possesses measure scores of zero across all measures. Intuition confirms this result since the current weight scheme does not require any changes to rank the alternative at the top. Alternatives without measure values due to infeasibility in the mathematical programs imply an insensitive alternative. The insensitive alternative can never rank on top regardless of the attribute weighting scheme.

A three-dimensional bar graph representing the measure values across all alternatives and measures reveals the sensitive alternatives. For the proposed alternatives in this research, the graph will indicate a clear break in relative measure values across the alternatives, thereby separating the alternatives into two groups, sensitive and not sensitive. Alternatives falling into the sensitive group across all five measures indicate sensitivity and require further examination. Additional resources applied to these sensitive alternatives will provide a refined perspective to the decision maker.

III.L. Summary

This chapter presented the methodology used in this research. Affinity diagramming organizes objectives into a hierarchical structure in the absence of a decision maker. The hierarchy serves as the basis for the additive value model. Once fully developed, alternatives are developed, scored, and ranked. Robust sensitivity analysis then reveals alternatives sensitive to the initial weighting scheme. The next chapter applies this methodology to the notional stability operations problem based on General Petraeus's September 2007 report to Congress.

IV. Results and Analysis

IV.A. Introduction

This chapter presents the results of the research conducted using the methodology described in the previous chapter. After developing the ranking of the alternatives, one-way sensitivity analysis shows a relatively insensitive value model. The use of five robust sensitivity measures, however, indicates alternatives are in fact sensitive to simultaneous changes in the weights.

IV.B. COA Scoring

After developing the value model and constructing alternative courses of action with their respective evaluation measures, the analysis begins. The first step uses the SDVFs to translate raw evaluation measures to values. The following example shows this translation for the *# Insurgents Crossing* measure of the *Status Quo* alternative. An estimated 50 insurgents cross the border and enter Iraq every month in the *Status Quo* alternative. This measure has a most preferred level of 0 and a least preferred level of 100. A decreasing exponential curve, with a ρ value of -14.4475 defines the SDVF. Based on Equation 3.3b, the *Status Quo* alternative has a raw score of 50 and a value for *# Insurgents Crossing* of 0.03045 as seen in Equation 4.1.

$$v(50) = \frac{1 - e^{\frac{-(100-50)}{-14.4475}}}{1 - e^{\frac{-(100-50)}{-14.4475}}} = 0.03045 \qquad \text{Equation 4.1}$$

Translating the remaining evaluation measures to value for each course of action occurs in a similar manner using the respective SDVFs. Table 4.1 shows the translated notional values for all alternatives.

Table 4.1 COA Evaluation Values

	% Coalition Lost	# Insurgents Crossing	Ethno-Sectarian Violence Death Rate	Non-Sectarian Violent Death Rate	Avg. hrs./day
Status Quo	0.990	0.030	0.112	0.877	0.852
Expel AQI	0.906	0.250	0.566	0.980	0.852
Train ISI	0.999	0.125	0.566	0.980	0.951
Institute Draft	0.953	0.015	0.000	0.038	0.500
Partial Coalition Withdrawal	0.997	0.001	0.112	0.326	0.720
Full Coalition Withdrawal	1.000	0.000	0.000	0.004	0.155
Pursue PKK	0.077	0.003	0.000	0.000	0.913
Self-Sustained Agriculture	0.990	0.015	0.412	0.877	0.852
24 hr Electricity Established	0.990	0.062	0.566	0.877	1.000
Lower Unemployment	0.990	0.043	0.412	0.877	0.852

	L H2O/person/day	L Fuel/person/day	Tons of Supplies Needed	Estimated # of Members	Willingness
Status Quo	0.001	0.320	0.000	0.950	1.000
Expel AQI	0.132	0.500	0.000	0.986	1.000
Train ISI	0.634	0.450	0.000	0.986	1.000
Institute Draft	0.000	0.200	0.000	0.995	0.000
Partial Coalition Withdrawal	0.000	0.300	0.000	0.857	0.500
Full Coalition Withdrawal	0.000	0.150	0.000	0.380	0.000
Pursue PKK	0.024	0.250	0.000	0.914	0.500
Self-Sustained Agriculture	0.004	0.370	0.970	0.962	1.000
24 hr Electricity Established	0.997	0.700	0.128	0.972	1.000
Lower Unemployment	0.924	0.400	0.500	0.980	0.500

	# of Units at Level I	# of Units at Level II	# of Units at Level III	# of Units at Level IV
Status Quo	0.060	0.585	0.338	0.513
Expel AQI	0.075	0.575	0.356	0.560
Train ISI	0.250	0.550	0.680	0.865
Institute Draft	0.060	0.585	0.275	0.419
Partial Coalition Withdrawal	0.060	0.785	0.582	0.142
Full Coalition Withdrawal	0.060	0.950	0.773	0.030
Pursue PKK	0.075	0.575	0.356	0.560
Self-Sustained Agriculture	0.060	0.585	0.338	0.513
24 hr Electricity Established	0.060	0.585	0.338	0.513
Lower Unemployment	0.060	0.585	0.338	0.513

This research then proceeds to find an overall value score for each COA. Based on the additive value function shown in Equation 3.5, the *Status Quo* COA has a value score of 0.451 on a scale from 0 to 1 as seen in Equation 4.2 below.

$$\begin{aligned} V^{StatusQuo} &= .215(.990) + .107(.030) + .268(.112) + .188(.877) \\ &+ .008(.852) + .011(.001) + .004(.320) + .142(0) + .008(.950) + .016(1) \\ &+ .016(.060) + .008(.585) + .003(.338) + .003(.338) + .003(.513) = .451 \end{aligned} \quad \text{Equation 4.2}$$

The value score then shows relative comparisons to the other alternatives. Table 4.2 shows the calculated value score for each COA and the next section discusses their rank order.

Table 4.2 COA Value Scores

	Value Score
Self-Sustained Agriculture	0.668
Train ISI	0.618
24 hr Electricity Established	0.608
Lower Unemployment	0.607
Expel AQI	0.600
Status Quo	0.451
Partial Coalition Withdrawal	0.338
Institute Draft	0.234
Full Coalition Withdrawal	0.214
Pursue PKK	0.050

IV.C. COA Ranking

Ranking the evaluated courses of action in descending order based on value provides an ordered list of COAs from most preferred to least preferred. An optimal course of action would have a value of 1 for each attribute and result in a value score of 1. The *Ideal* COA shown in Figure 4.1 provides a visual reference for comparison. As indicated by Figure 4.1, the *Self-Sustained Agriculture* alternative is the recommended course of action with a value score of 0.668. The next four alternatives, *Train ISI*, *24 hr Electricity Established*, *Lower Unemployment*, and *Expel AQI* all score relatively close in value and additionally are not far from *Self-Sustained Agriculture*. In an uncertain environment, it is possible that if weights or evaluation measures are imprecise, then a different course of action may be the preferred alternative. This is where sensitivity analysis begins and is the focus of the remainder of this research.

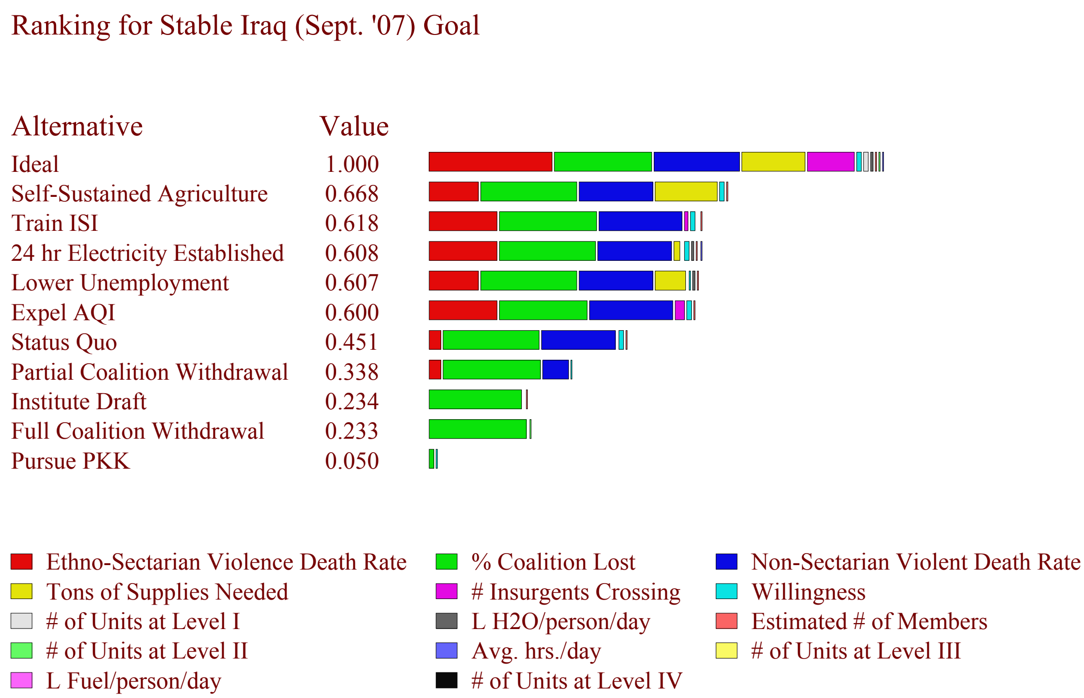


Figure 4.1 Courses of Action Rank Order

IV.D. Traditional Sensitivity Analysis

One-way sensitivity analysis can indicate the alternatives sensitive to small changes in a single weight. For the purposes of this research, a change in weight less than 0.05 resulting in a different alternative being preferred indicates sensitive alternatives and the influence weights. In the problem addressed by this research, a change of 0.05 in a single weight is a 20% change in weight if considering *Ethno-Sectarian Violence Death Rate*, the attribute with the highest weight. This same change of 0.05, if looking at *Tons of Supplies Delivered,* is a 1250% change in weight. Most of the attributes have weights similar to that of *Tons of Supplies Delivered*. While a decision maker may provide imprecise weights for attributes, they should not be off by more than 20%. One-way sensitivity analysis, however, can only show this analysis one

attribute weight at a time. For example, Figure 4.2 shows the sensitivity graph for the measure associated with *Estimated Insurgent Border Crossings*. The weight associated with this measure would have to increase from 0.107 to greater than 0.3 in order for *Expel AQI* to become the preferred alternative. This graph does not indicate one-way sensitivity in the model.

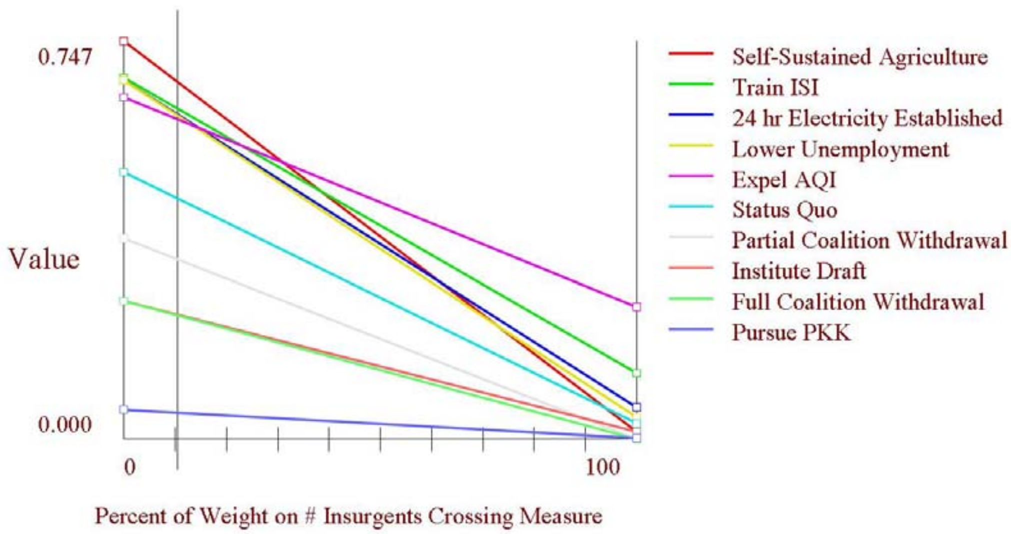


Figure 4.2 *Estimated Insurgent Border Crossings* Sensitivity Graph

Further examination shows that all attributes require a change in weight of 0.05 or greater to cause the preferred alternative to change. The changes needed in the *Potable Water* and *Humanitarian Relief* weights to change the preferred alternative are the closest to 0.05 as seen by Figures C.7 and C.8 in Appendix C, but are greater than 0.05. *Potable Water* has a weight of 0.011, a change of 0.05 is at 455% change; while *Humanitarian Relief's* weight of 0.142 changing by 0.05 is a 35% change. Therefore, these attributes do not indicate weight sensitivity in the model. This means that the decision model is not weight sensitive based on traditional sensitivity analysis techniques. Appendix C shows the remainder of the one-way sensitivity graphs. While a change in one attribute weight

at a time does not indicate sensitivity, it is possible that multiple small changes in attribute weights could indicate sensitivity.

IV.E. Robust Sensitivity Measures

Systematically changing all weights at the same time presents a challenge due to the infinite number of possible weight combinations. The sensitivity measures discussed in the previous chapter provide structure to finding different combinations of weights that could indicate sensitivity or insensitivity in a value model. The intent of all the sensitivity measures is the same; they indicate the distance between the assessed weights from Table 3.3 and a new set of weights that causes a course of action to become the preferred alternative, regardless of its original rank. While one-way sensitivity analysis examines sensitivity one weight at a time, the sensitivity measures used here examine one alternative at a time. The top-ranked alternative satisfies all constraints used to calculate the new weights. Therefore, the top ranked alternative, *Self-Sustained Agriculture*, has a sensitivity measure of zero. If the mathematical program cannot find a feasible solution to the problem by satisfying all constraints, then the course of action examined is considered insensitive. An insensitive course of action cannot become the preferred alternative regardless of the weights chosen, e.g. *Pursue PKK* course of action. Therefore, it is an insensitive alternative, and it is not possible to calculate an associated measure. This implies the recommendation of removing the *Pursue PKK* alternative because it will never become the top ranked alternative regardless of the weights chosen for the fourteen attributes in the value model developed on 2007 conditions. Table 4.3 shows all five calculated measures for the ten alternative courses of action.

Table 4.3 Sensitivity Measures

Alternative	Least Squares	1-norm	∞-norm	2-norm	%	Comments
Status Quo	0.284	1.181	0.268	0.533	1636.80%	
Expel AQI	0.006	0.150	0.043	0.076	39.50%	
Train ISI	0.002	0.062	0.021	0.039	24.95%	
Institute Draft	0.447	1.504	0.474	0.668	5827.32%	
Partial Coalition Withdrawal	0.183	0.941	0.215	0.428	317.49%	
Full Coalition Withdrawal	0.120	0.737	0.192	0.346	273.08%	
Pursue PKK						(Insensitive Alternative)
Self-Sustained Agriculture	0	0	0	0	0.00%	(Ranked #1 by Assessed Weights)
24 hr Electricity Established	0.002	0.069	0.025	0.045	48.85%	
Lower Unemployment	0.003	0.099	0.040	0.055	119.55%	

Rank ordering these COAs in ascending order based on each measure reveals the same ranking as ranking the COAs in descending order according to the original value, except for the % measure. The % measure shows *Expel AQI* ranked third instead of fifth. This ranking irregularity stems from the fact that percent change in weight is not the same as the absolute change in weight. Tables 4.4 – 4.8 show the COAs in rank order according to the measures. There are comments located in the tables output from Premium Solver whose explanation follows each table.

Table 4.4 COAs Rank Ordered by the Least Squares Measure

Alternative	**Least Squares**	**Comments**
Self-Sustained Agriculture	0	(Ranked #1 by Initial Weights)
Train ISI	0.002	Solver found a solution. All constraints and optimality conditions are satisfied.
24 hr Electricity Established	0.002	Solver found a solution. All constraints and optimality conditions are satisfied.
Lower Unemployment	0.003	Solver found a solution. All constraints and optimality conditions are satisfied.
Expel AQI	0.006	Solver found a solution. All constraints and optimality conditions are satisfied.
Full Coalition Withdrawal	0.120	Solver found a solution. All constraints and optimality conditions are satisfied.
Partial Coalition Withdrawal	0.183	Solver found a solution. All constraints and optimality conditions are satisfied.
Status Quo	0.284	Solver found a solution. All constraints and optimality conditions are satisfied.
Institute Draft	0.447	Solver found a solution. All constraints and optimality conditions are satisfied.
Pursue PKK		(Insensitive Alternative)

The same message appeared following the calculation of the least squares measure for each alternative. Using the Standard GRG Nonlinear solver for these quadratic programs indicates an optimal solution that satisfies the Karush-Kuhn-Tucker (KKT) conditions for local optimality (Premium Solver V8, 2007:251). Knowing the problem is convex reveals that the local optimal is a global optimum.

Table 4.5 COAs Rank Ordered by the 1-norm Measure

Alternative	1-norm	Comments
Self-Sustained Agriculture	0	(Ranked #1 by Initial Weights)
Train ISI	0.062	Solver found a solution. All constraints and optimality conditions are satisfied.
24 hr Electricity Established	0.069	Solver found a solution. All constraints and optimality conditions are satisfied.
Lower Unemployment	0.099	Solver found a solution. All constraints and optimality conditions are satisfied.
Expel AQI	0.150	Solver found a solution. All constraints and optimality conditions are satisfied.
Full Coalition Withdrawal	0.737	Solver found a solution. All constraints and optimality conditions are satisfied.
Partial Coalition Withdrawal	0.941	Solver found a solution. All constraints and optimality conditions are satisfied.
Status Quo	1.181	Solver found a solution. All constraints and optimality conditions are satisfied.
Institute Draft	1.504	Solver found a solution. All constraints and optimality conditions are satisfied.
Pursue PKK		(Insensitive Alternative)

The message "Solver found a solution. All constraints and optimality conditions are satisfied," from the linear program indicates global optimality (Premium Solver V8, 2007:234). The linear program cannot find a combination of weights that will lower the value of the measure. A rapid computational time and global optimality make the *1-norm* an attractive measure for future studies.

Table 4.6 COAs Rank Ordered by the ∞-norm Measure

Alternative	∞-norm	Comments
Self-Sustained Agriculture	0	(Ranked #1 by Initial Weights)
Train ISI	0.021	Solver has converged to the current solution. All constraints are satisfied.
24 hr Electricity Established	0.025	Solver has converged to the current solution. All constraints are satisfied.
Lower Unemployment	0.040	Solver cannot improve the current solution. All constraints are satisfied.
Expel AQI	0.043	Solver has converged to the current solution. All constraints are satisfied.
Full Coalition Withdrawal	0.192	Solver has converged to the current solution. All constraints are satisfied.
Partial Coalition Withdrawal	0.215	Solver cannot improve the current solution. All constraints are satisfied.
Status Quo	0.268	Solver cannot improve the current solution. All constraints are satisfied.
Institute Draft	0.474	Solver cannot improve the current solution. All constraints are satisfied.
Pursue PKK		(Insensitive Alternative)

The Standard Evolutionary solver resulted in two different messages when solving for the infinity-norm measure. The most common message, "Solver cannot improve the current solution," occurs frequently with this solver because a heuristic cannot guarantee optimality (Premium Solver V8, 2007:236). This means the solver cannot improve the current solution even though it has not met conditions of convergence or optimality (Premium Solver V8, 2007:235). The other message, "Solver has converged to the current solution," indicates that solver has either found an optimal solution or the population has lost diversity, a problem common in genetic and evolutionary algorithms (Premium Solver V8, 2007:257). The problem prevents the algorithm from creating new and better solutions.

Table 4.7 COAs Rank Ordered by the 2-norm Measure

Alternative	2-norm	Comments
Self-Sustained Agriculture	0	(Ranked #1 by Initial Weights)
Train ISI	0.039	Solver found a solution. All constraints and optimality conditions are satisfied.
24 hr Electricity Established	0.045	Solver found a solution. All constraints and optimality conditions are satisfied.
Lower Unemployment	0.055	Solver found a solution. All constraints and optimality conditions are satisfied.
Expel AQI	0.076	Solver found a solution. All constraints and optimality conditions are satisfied.
Full Coalition Withdrawal	0.346	Solver found a solution. All constraints and optimality conditions are satisfied.
Partial Coalition Withdrawal	0.428	Solver found a solution. All constraints and optimality conditions are satisfied.
Status Quo	0.533	Solver found a solution. All constraints and optimality conditions are satisfied.
Institute Draft	0.668	Solver found a solution. All constraints and optimality conditions are satisfied.
Pursue PKK		(Insensitive Alternative)

The message "Solver found a solution. All constraints and optimality conditions are satisfied," for these quadratic programs indicates an optimal solution that satisfies the Karush-Kuhn-Tucker (KKT) conditions for local optimality (Premium Solver V8, 2007:251). Knowing the problem is convex reveals that the local optimal is a global optimum. The measure values for the 2-norm are the square root of the least squares measure values for every alternative. The 2-norm, however, is favorable compared to the least squares measure because it allows for better visual differentiation between the courses of action when plotted on a bar graph. The computational time between the two is comparable.

Table 4.8 COAs Rank Ordered by the % Measure

Alternative	%	Comments
Self-Sustained Agriculture	0.00%	(Ranked #1 by Initial Weights)
Train ISI	24.95%	Solver cannot improve the current solution. All constraints are satisfied.
Expel AQI	39.50%	Solver cannot improve the current solution. All constraints are satisfied.
24 hr Electricity Established	48.85%	Solver cannot improve the current solution. All constraints are satisfied.
Lower Unemployment	119.55%	Solver cannot improve the current solution. All constraints are satisfied.
Full Coalition Withdrawal	273.08%	Solver cannot improve the current solution. All constraints are satisfied.
Partial Coalition Withdrawal	317.49%	Solver cannot improve the current solution. All constraints are satisfied.
Status Quo	1636.80%	Solver cannot improve the current solution. All constraints are satisfied.
Institute Draft	5827.32%	Solver cannot improve the current solution. All constraints are satisfied.
Pursue PKK		(Insensitive Alternative)

The Standard Evolutionary solver resulted in the message, "Solver cannot improve the current solution," which occurs frequently with this solver because a heuristic cannot guarantee optimality (Premium Solver V8, 2007:236). This means the solver cannot improve the current solution even though it has not met conditions of convergence or optimality (Premium Solver V8, 2007:235).

IV.F. Robust Sensitivity Analysis

The calculation of the measures enables robust sensitivity analysis of the alternatives. Figure 4.3 shows a graphical representation of the calculated measures. It shows a large relative change in calculated sensitivity measure between the top five alternatives and the bottom four from Figure 4.2. The small deltas for the top five

measures indicate sensitivity and that these alternatives require further examination. The rank order of these measure values is the same for all measures except for the *% Measure*.

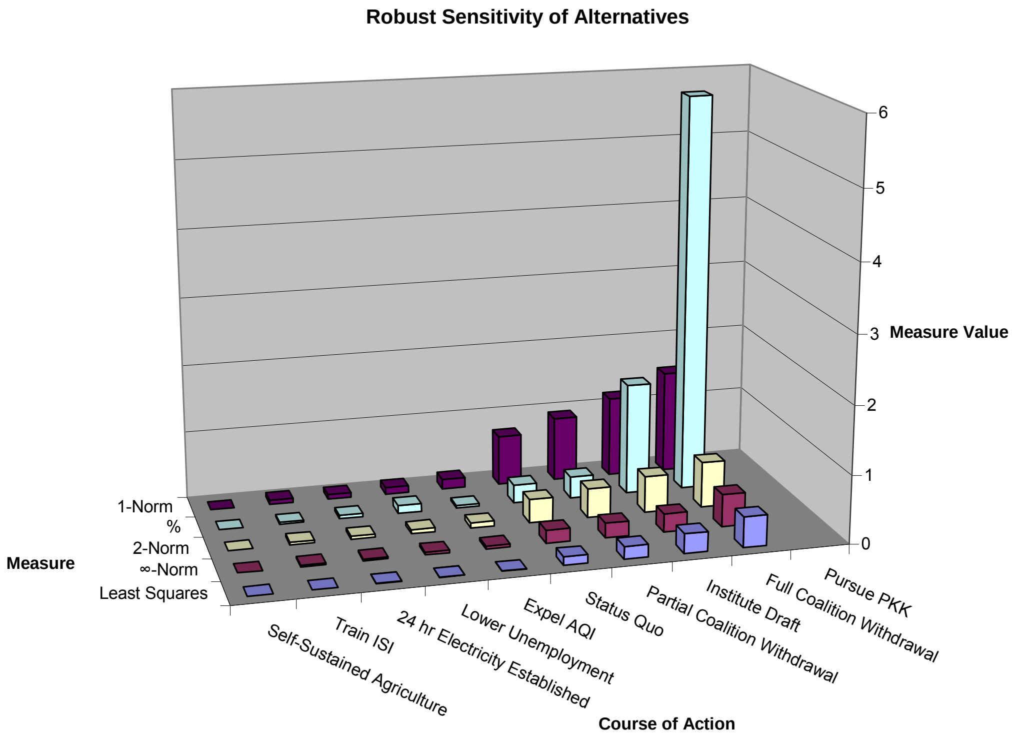


Figure 4.3 Robust Sensitivity Analysis

The *% Measure* may p. The other measures examine the absolute distance between the assessed weights and the new calculated weights. The *% Measure* examines the percent change for each weight. According to this measure, a change in weight from 0.01 to 0.02 and a change from 0.1 to 0.2 are identical with a 100% change. If a weight changes from 0.01 to 0.02 by the other measures it is sensitive, while a change from 0.1 to 0.2 does not indicate sensitivity.

Since the four other measures indicate the same rank order, the recommended measure is the *1-norm*. The *1-norm* measure has the shortest computational time and

only changes a few weights. The short computational time allows an analyst to conduct rapid sensitivity analysis in which the commander requires a timely response. A few weight changes, as opposed to all the weights changing, could allow for easier interpretation of the weight changes since it is sometimes a trade of weight from one or two attributes to another. Table 4.9 shows that the *1-norm* measure exchanges weight between *L H20/person/day* and *Tons of Supplies Needed* in order to make *Train ISI* the top alternative. The linear program transfers the weight from *Tons of Supplies Needed* to *L H2O/person/day* to give *Train ISI* the highest value of all alternatives.

Table 4.9 Absolute Change in Weights by Measures for *Train ISI*

		Abs. Δ Between Original and Calculated Weights				
Train ISI	Original Weight	Least Squares	1-norm	∞-norm	2-norm	%
% Coalition Lost	0.215	0.002	0	0.019	0.002	0.053
# Insurgents Crossing	0.107	0.001	0	0.002	0.001	0.006
Ethno-Sectarian Violence Death Rate	0.268	0.002	0	0.006	0.002	0.067
Non-Sectarian Violent Death Rate	0.188	0.001	0	0.018	0.001	0.025
Avg. hrs./day	0.008	0.001	0	0.007	0.001	0.002
L H2O/person/day	0.011	0.017	0.031	0.021	0.017	0.003
L Fuel/person/day	0.004	0.000	0	0.000	0.000	0.001
Tons of Supplies Needed	0.142	0.033	0.031	0.021	0.033	0.036
Estimated # of Members	0.008	0.002	0	0.008	0.002	0.001
Willingness	0.016	0.002	0	0.003	0.002	0.004
# of Units at Level I	0.016	0.004	0	0.020	0.004	0.004
# of Units at Level II	0.008	0.004	0	0.008	0.004	0.002
# of Units at Level III	0.005	0.008	0	0.021	0.008	0.001
# of Units at Level IV	0.003	0.009	0	0.021	0.009	0.001

The *2-norm* and *Least Squares* measures result in the calculation of identical new weights for all courses of action. The quadratic programs for these two measures run

slightly slower than the linear programs for the *1-norm*, additionally, they cannot guarantee global optimality and they change all weights. Without global optimality ensured, the measure could be greater than it would be at optimality and indicate an alternative as insensitive and remove it from further consideration when it was in fact a viable alternative that needs further examination. When all weights change, the interpretations of these changes prove difficult and time consuming. It is not as easy to see how the weight moved from one attribute to another as seen with the *1-norm* measure and time is often not a luxury when trying to select a course of action. The quadratic programs changing all the weights sometime result in a weight changing on a scale of 0.001 while another weight may change on a scale of 0.1.

For example, Table 4.10 shows the original weights and then the absolute change in the weights required to raise the sixth ranked alternative, *Status Quo*, to make it the preferred alternative. When examining the *Least Squares* measure, one can see that the weight for # *of Units at Level IV* changes by 0.003 while the weight for # *of Units at Level II* changes by 0.309. This demonstrates two orders of magnitude difference between the changes in the two weights. The change in weight on a scale of 0.1 contributes the most to the alternative's change in ranking, while weight changes on the scale of 0.001 do not and could be artifacts of the measure optimization.

Table 4.10 Absolute Change in Weights by Measures for *Status Quo*

		Abs. Δ Between Original and Calculated Weights				
Status Quo	Original Weight	Least Squares	1-norm	∞-norm	2-norm	%
% Coalition Lost	0.215	0.176	0.000	0.264	0.176	0.361
# Insurgents Crossing	0.107	0.107	0.107	0.107	0.107	0.107
Ethno-Sectarian Violence Death Rate	0.268	0.268	0.268	0.268	0.268	0.268
Non-Sectarian Violent Death Rate	0.188	0.113	0.017	0.186	0.113	0.188
Avg. hrs./day	0.008	0.008	0.008	0.008	0.008	0.008
L H2O/person/day	0.011	0.011	0.011	0.011	0.011	0.011
L Fuel/person/day	0.004	0.004	0.004	0.004	0.004	0.004
Tons of Supplies Needed	0.142	0.142	0.142	0.142	0.142	0.142
Estimated # of Members	0.008	0.008	0.008	0.008	0.008	0.008
Willingness	0.016	0.202	0.041	0.258	0.202	0.266
# of Units at Level I	0.016	0.016	0.016	0.015	0.016	0.016
# of Units at Level II	0.008	0.309	0.550	0.225	0.309	0.133
# of Units at Level III	0.005	0.005	0.005	0.003	0.005	0.005
# of Units at Level IV	0.003	0.003	0.003	0.001	0.003	0.003

The measures calculated using non-linear programming, the *∞-norm* and *% measures*, require more time than the quadratic programming measures, and once again cannot guarantee optimality and change all the weights. The changes across all weights for these measures show the same characteristics shown by the *2-norm* and *Least Squares* measures. Again, the small change in weights could be artifacts of the measure optimization.

One-way sensitivity analysis offers a way to examine the sensitivity of alternatives and attribute weights. The robust sensitivity analysis approach proposed here examines the decision sensitivity to the alternatives. Sensitive weights guide a decision maker to further examine the attributes associated with those weights. This further

examination could include placing more resources into soliciting the weight for the attribute or ensuring the evaluation measures are correct for the alternatives.

A sensitive weight should have a common trait across all alternatives that indicates its sensitivity. In one-way sensitivity analysis, a required small change in weight indicates a sensitive alternative and weight while a large change indicates an insensitive alternative and weight. Does this same rule apply to n-way sensitivity analysis or does the opposite apply? Does a required large change in weight indicate sensitivity since small changes could be artifacts of the measure optimization? Even if the author knew the answer to this question, further examination could not find a universal trait across all alternatives for a weight in order to identify a specific weight or group of weights as sensitive. Figure 4.4 shows that sensitive alternatives previously identified shows the weights for *L H20/person/day* and *Tons of Supplies Delivered* change the most for sensitive alternatives, but these weights change the least for non-sensitive alternatives.

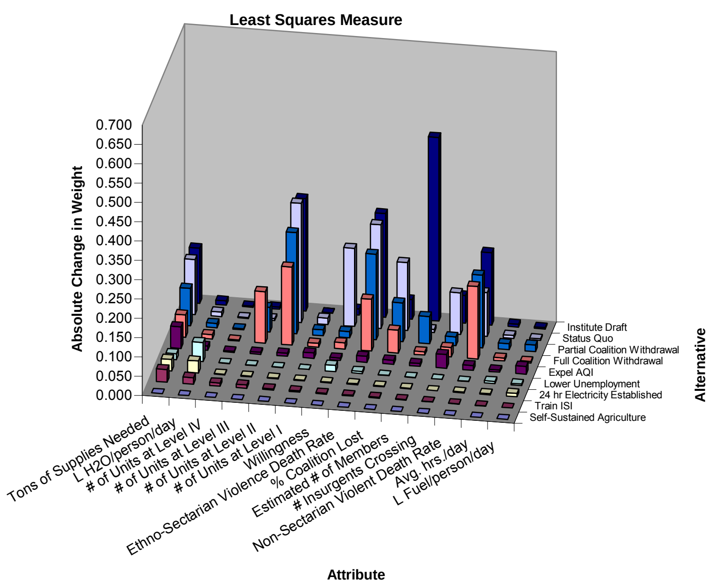


Figure 4.4 Absolute Change in Weights for *Least Squares* Measure

Perhaps, if the absolute change in a weight ranks in the top five, or bottom five, for all alternatives, then it indicates sensitivity. Closer examination found that at times, a weight will remain in one of these categories across all alternatives, but this is not universal across all measures. Again, even if it was universal, is a ranked weight change in the top or bottom five classified as sensitive? Additional analysis included examining the new rank order of weights based on magnitude compared to the original order. Appendix D shows some of the work conducted in an attempt to determine the sensitivity of the attribute weights for one alternative in a robust sensitivity analysis.

IV.G. Summary

This chapter stepped through the analysis of alternative courses of action from a notional value model used to evaluate stability operations in Iraq. The focus remained on the sensitivity analysis of the model. The one-way sensitivity analysis did not indicate sensitivity within the model. The proposed robust sensitivity analysis approach, however, indicated that five of the ten alternatives are sensitive when multiple weights change as opposed to only one. Instead of calculating all five robust sensitivity measures, the *1-norm* is the recommended measure. This analysis best supports the initial planning phases of military operations and again when the nature of the problem faced changes.

V. Discussion

V.A. Introduction

Today's United States military pursues effects based operations and predictive battlespace awareness. If the military can predict an adversary's plans or can predict how an adversary will react to actions by the United States, then the United States military can increase its effectiveness. This is not a new concept. Military commanders have recognized the need for this capability at least as early as Sun Tzu in the sixth century B.C. In the face of this capability gap, commanders desire implementable courses of action that are robust to uncertainty.

One approach to assessing courses of action used in today's military is additive value models. The problem lies within the solicitation of weights. The weights are subjective assessments of importance between the attributes in the model. When evaluating courses of action, an ideal course of action dominates all others. A dominant alternative scores better in every evaluation measure compared to other alternatives. If a course of action is dominant, then the value model is insensitive. There is rarely a dominant course of action to select.

If there is not a dominant solution, then the commander desires a robust alternative that is relatively insensitive and still meets his/her objectives. Insensitive alternatives indicate a required large change in the weights in order for those alternatives to become the preferred alternative. If a large change in weights is required to cause a change, then the assessed weights can be slightly incorrect and the recommended decision will not change.

V.B. Research Contributions

This research contributes to the field of sensitivity analysis for additive value models. It not only compares the results from three previously published sensitivity measures, but it also proposes a new sensitivity measure that is not recommended. These sensitivity measures enable a more robust approach to sensitivity than the standard one-way sensitivity analysis. Instead of varying one weight at time, the robust approach using sensitivity measures allows for the interaction of weight changes. This approach can indicate sensitive alternatives requiring further examination.

V.C. Recommendations for Further Research

After discovering sensitive alternatives, closer examination should occur on sensitive attributes. One-way sensitivity analysis indicates sensitive attributes and then indicates the alternatives sensitive to that attribute. This approach to robust sensitivity analysis works in the opposite order. First, it identifies sensitive alternatives. Then, it would identify the sensitive attributes. Determining these sensitive attributes is an avenue for future research. Traditional sensitivity analysis and the robust sensitivity analysis for alternatives indicate sensitivity based on small changes in weights. When trying to determine the sensitive attributes using the robust sensitivity approach, does a required small or large change in an attribute weight indicate sensitivity? Another avenue includes testing these measures on several additive value models to find consistency. The notional test case in this research was intentionally insensitive on all attributes according to one-way sensitivity analysis. How does the outcome of the sensitivity measures change when there is an indication of sensitivity in the model based

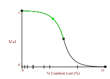
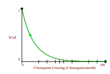
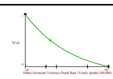
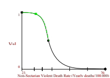
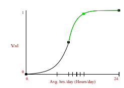
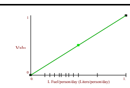
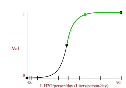
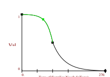
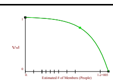
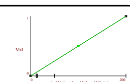
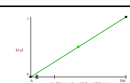
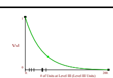
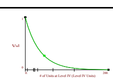
on one attribute? An additional avenue for further research includes examining the sensitivity of a decision based on the SDVFs and evaluation measures.

V.D. Conclusions

Robust sensitivity analysis using sensitivity measures provides a more realistic approach to sensitivity analysis for additive value models in the planning stage. The world constantly changes, and generally does not change only one attribute at a time. Instead, multiple attributes change simultaneously. The robust sensitivity measures attempt to address this situation.

Appendix A. Evaluation Measure Definitions

Table A.1 Evaluation Measure Summary

Evaluation Measure	SDVF	Most Preferred	Least Preferred	Units
% Coalition Lost		0	50	%
# Insurgents Crossing		0	100	# Insurgents/month
Ethno-Sectarian Violence Death Rate		10	50	Yearly deaths/100,000
Non-Sectarian Violent Death Rate		23	70	Yearly deaths/100,000
Avg. hrs./day		24	0	Hours/day
L fuel/person/day		1	0	Liters/person/day
L H20/person/day		90	45	Liters/person/day
Tons of Supplies Needed		0	270	Tons of food
Estimated # of Members		0	100,000	People
Willingness		High	Low	Categorical
# of Units at Level I		200	0	ISI Units
# of Units at Level II		200	0	ISI Units
# of Units at Level III		0	200	ISI Units
# of Units at Level IV		0	200	ISI Units

Level I: Fully Independent (Petraeus, 2007:12)
Level II: Iraqi Lead with Coalition Support (Petraeus, 2007:12)
Level III: Fighting Side by Side (Petraeus, 2007:12)
Level IV: Unit Forming (Petraeus, 2007:12)

Appendix B. Single Dimensional Value Functions

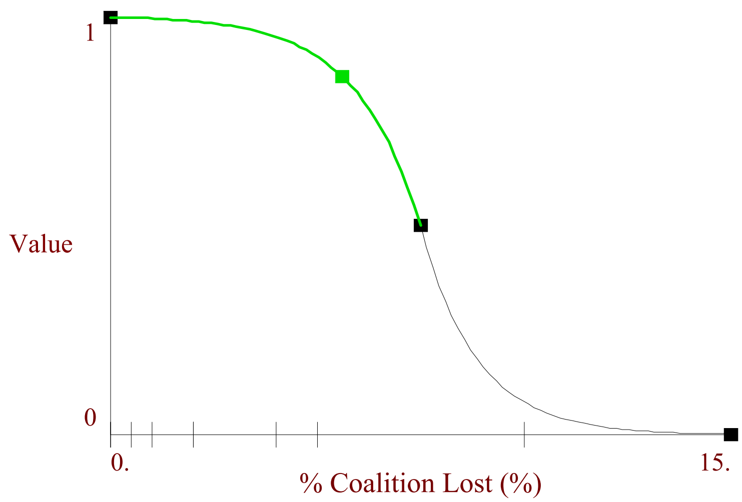


Figure B.1 Coalition Forces' Safety SDVF

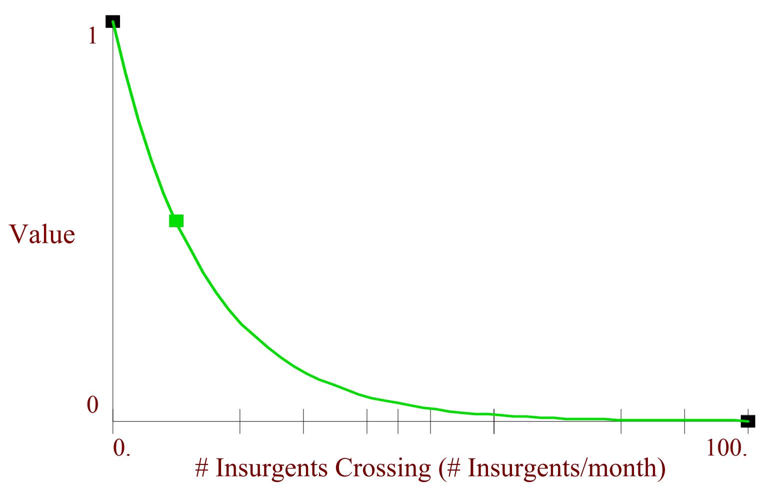


Figure B.2 Estimated Insurgent Border Crossings SDVF

Figure B.3 Ethno-Sectarian Violence SDVF

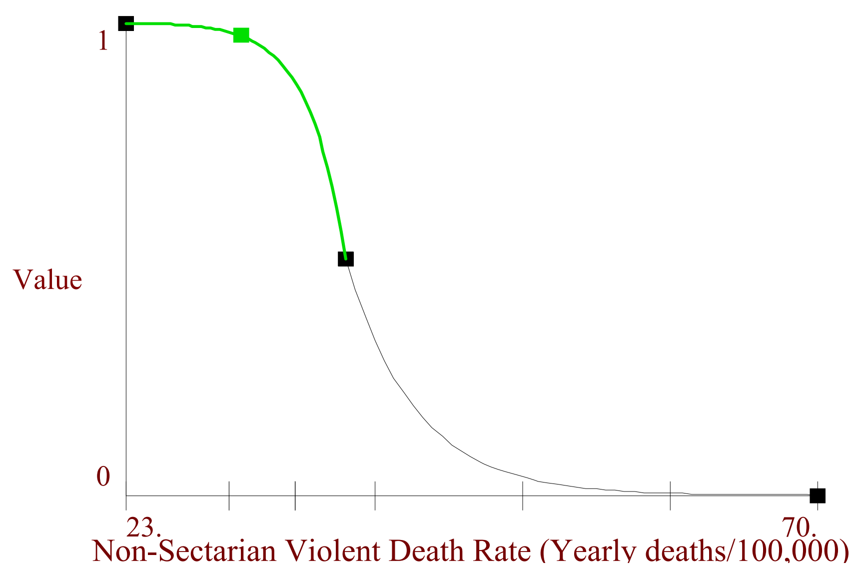


Figure B.4 Non-Sectarian Violence SDVF

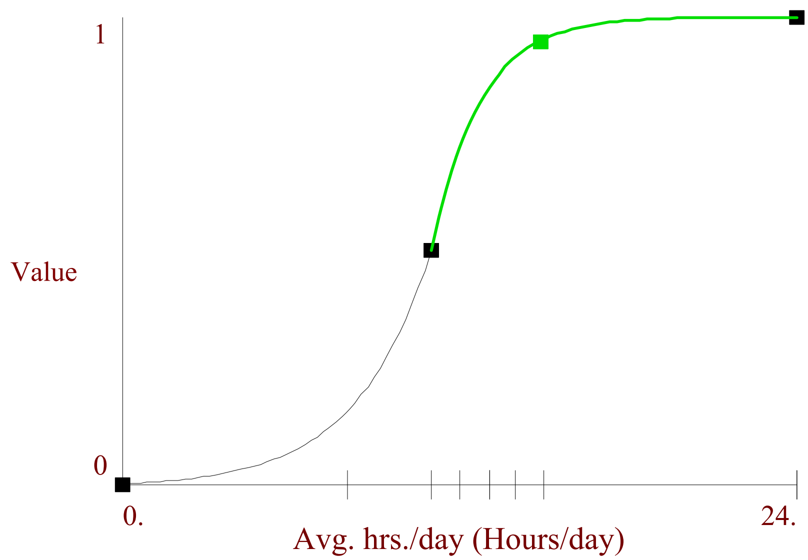


Figure B.5 Electricity SDVF

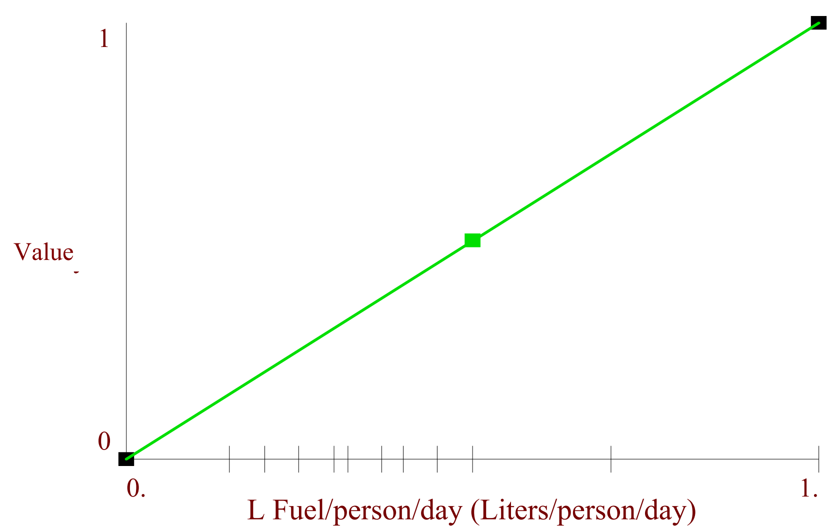


Figure B.6 Fuel (Heating/Cooking) SDVF

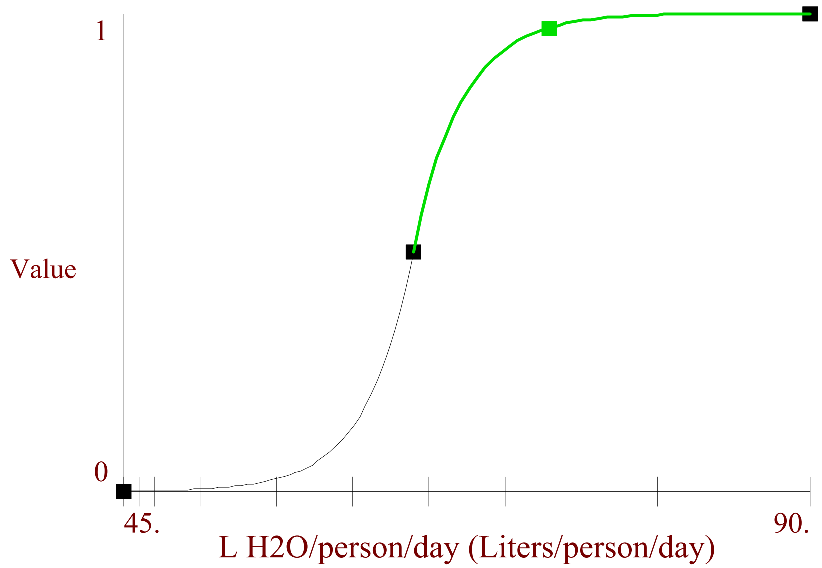


Figure B.7 Potable Water SDVF

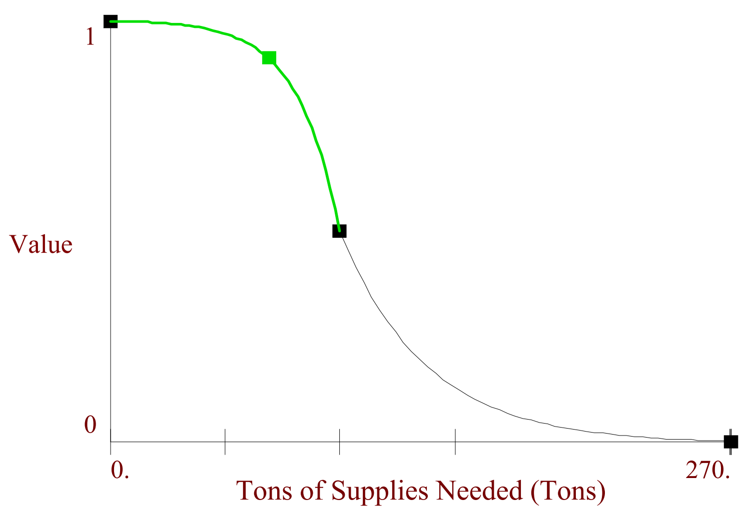


Figure B.8 Humanitarian Relief SDVF

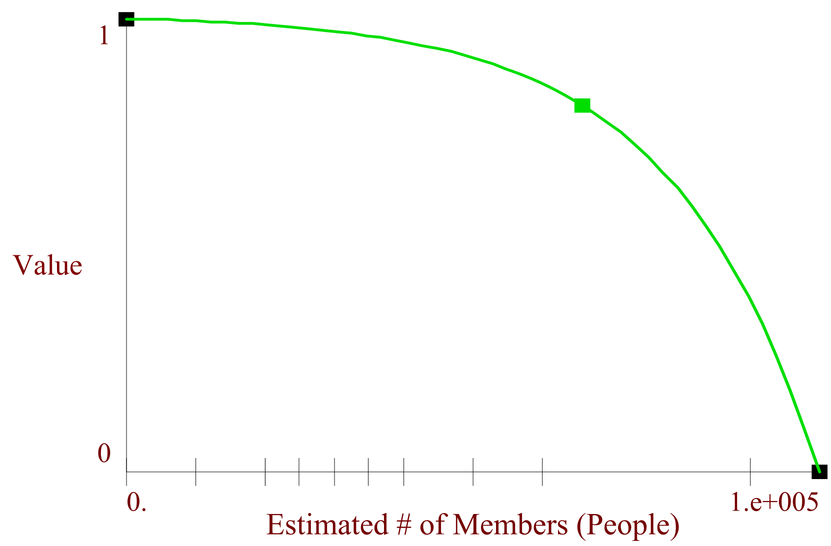


Figure B.9 Additional People Needed for ISI SDVF

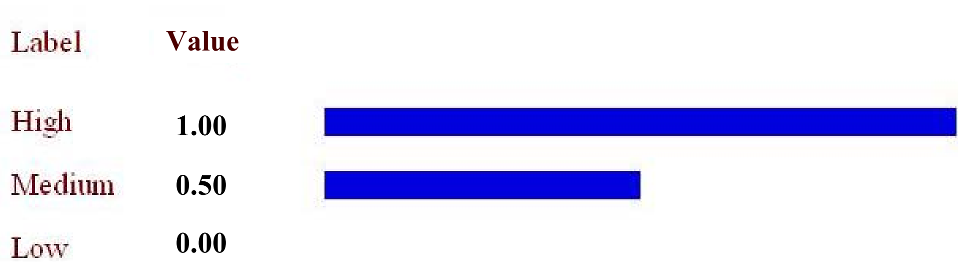


Figure B.10 Locals' Willingness to Serve SDVF

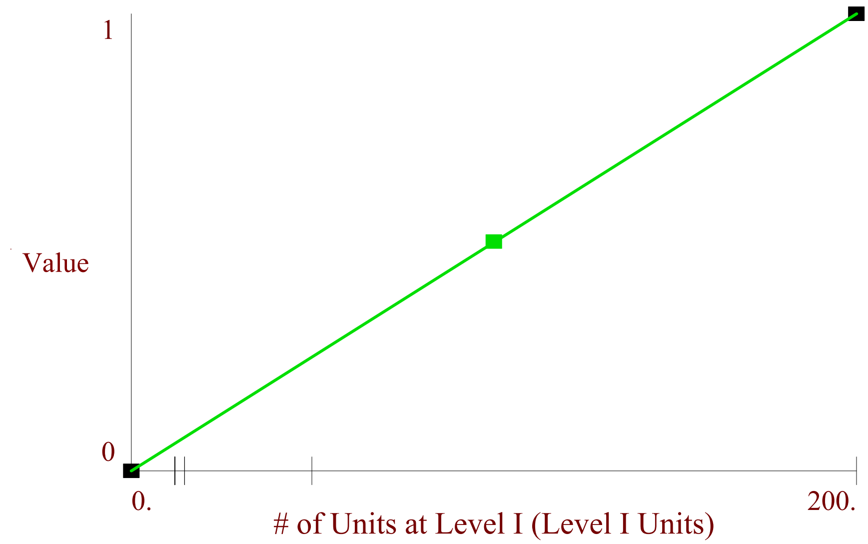


Figure B.11 Capability Level I SDVF

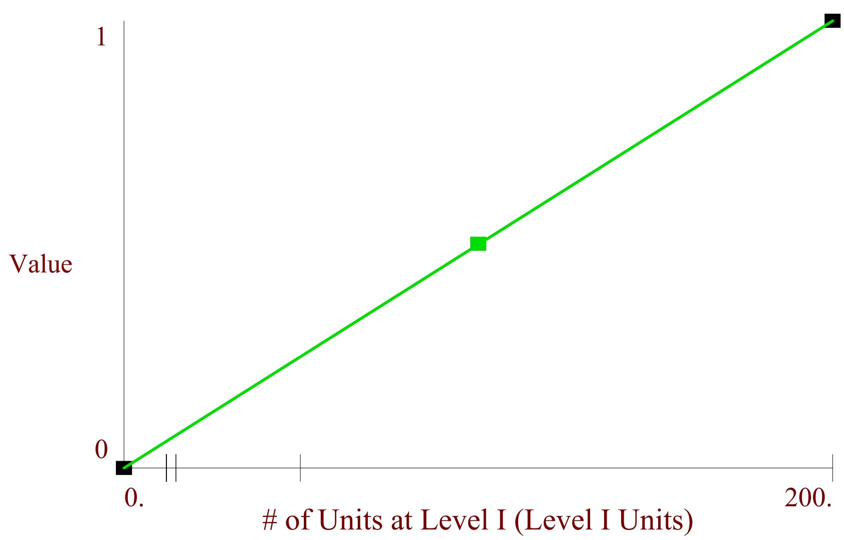


Figure B.12 Capability Level II SDVF

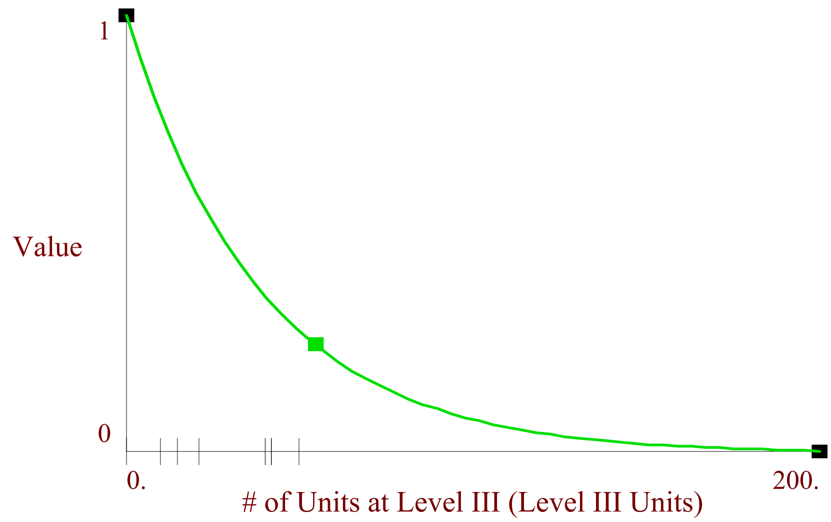


Figure B.13 Capability Level III SDVF

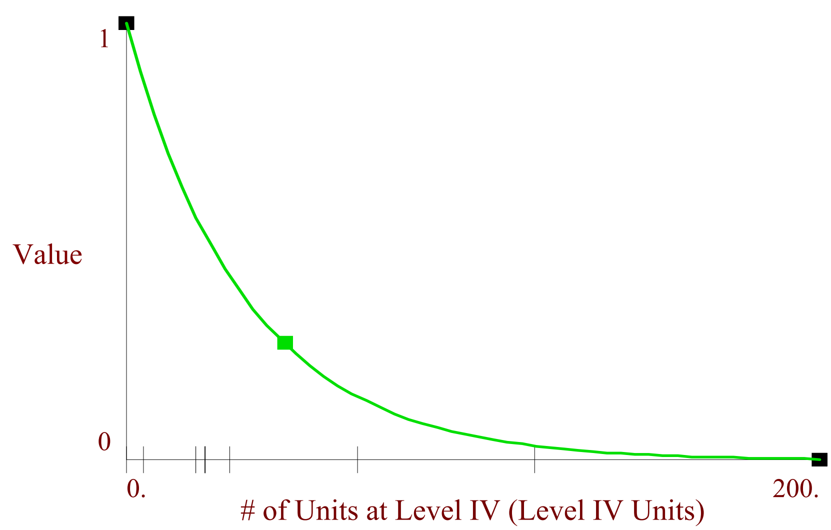


Figure B.14 Capability Level IV SDVF

Appendix C. One-Way Sensitivity Analysis

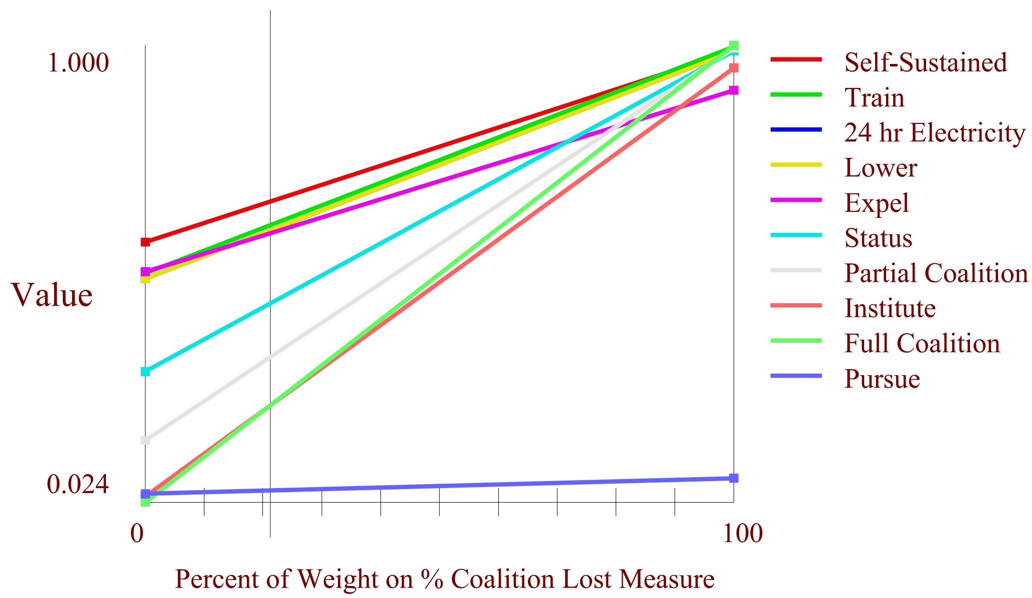


Figure C.1 Coalition Forces' Safety One-Way Sensitivity Graph

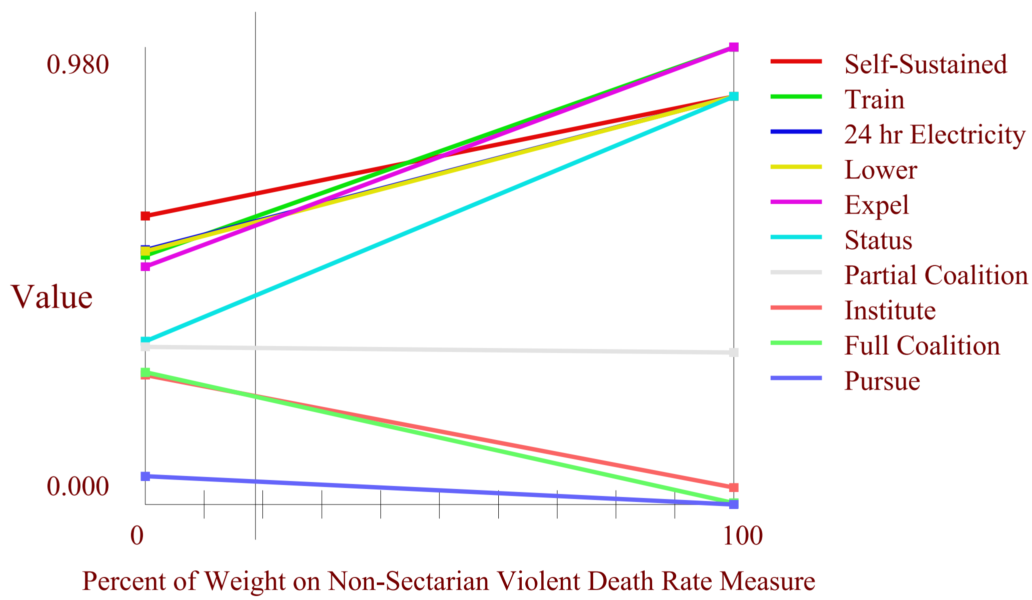


Figure C.2 Estimated Insurgent Border Crossings One-Way Sensitivity Graph

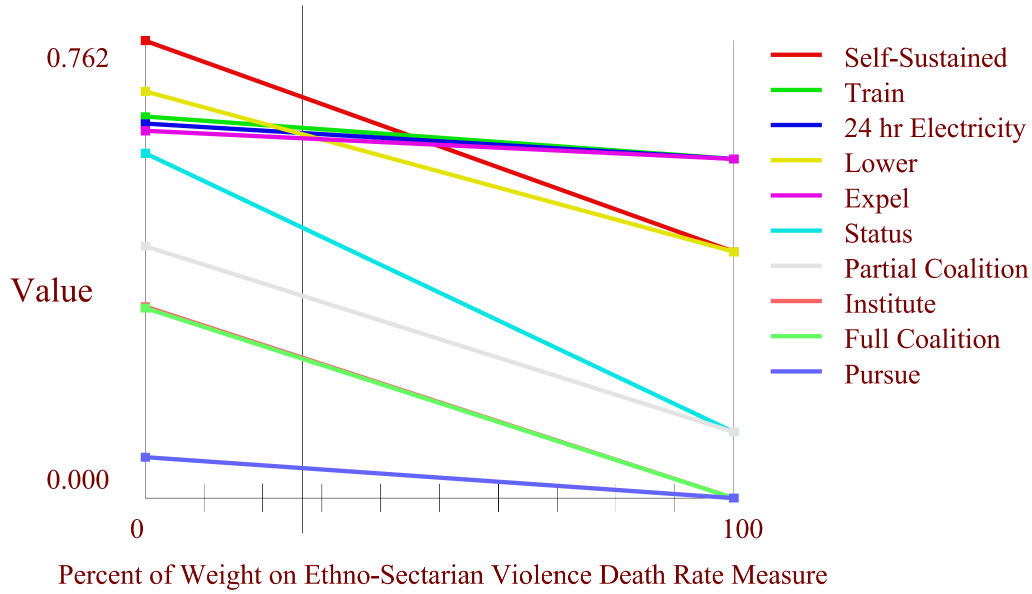


Figure C.3 Ethno-Sectarian Violence One-Way Sensitivity Graph

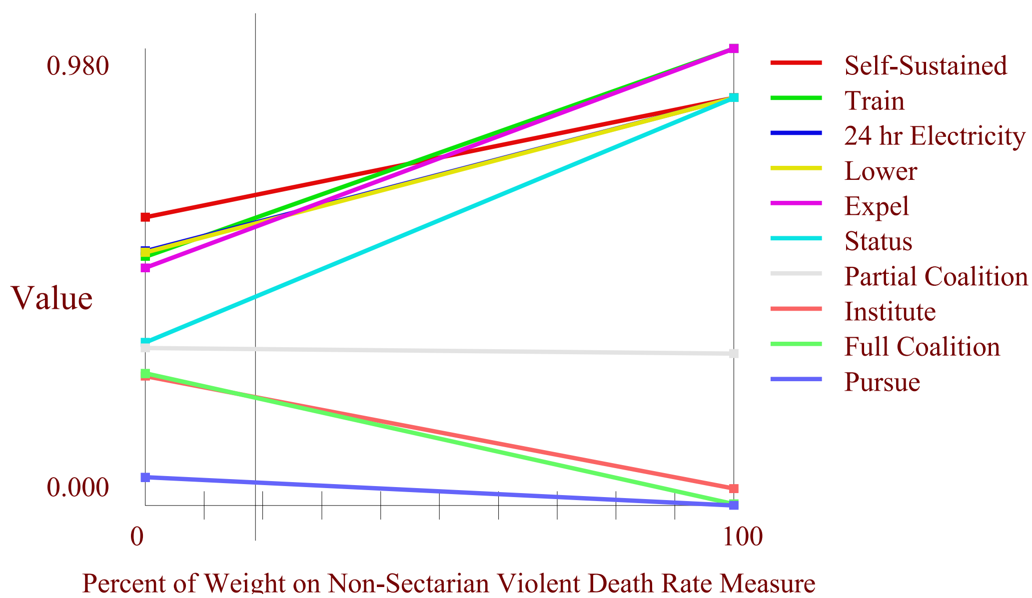


Figure C.4 Non-Sectarian Violence One-Way Sensitivity Graph

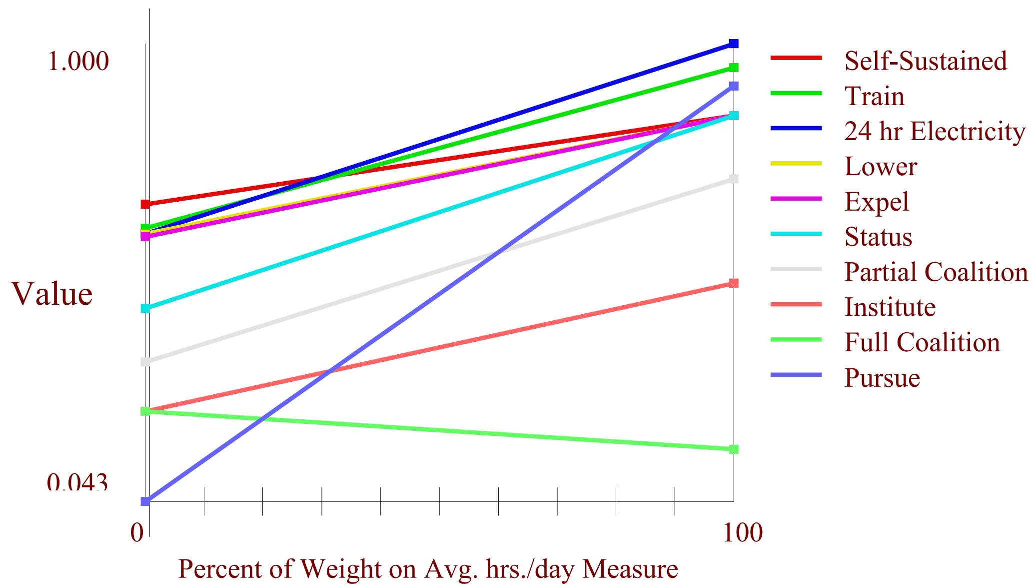


Figure C.5 Electricity One-Way Sensitivity Graph

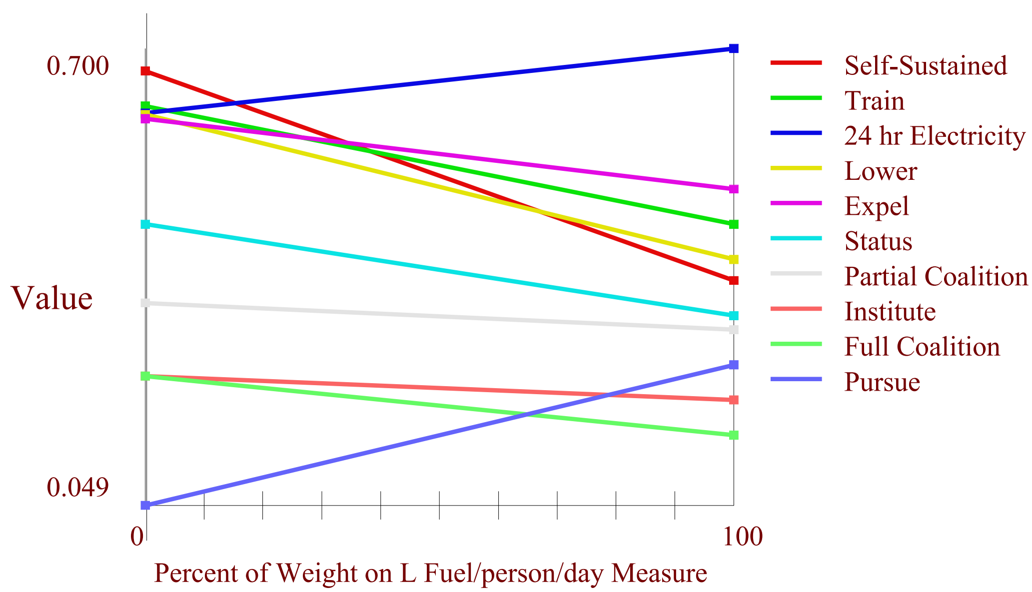


Figure C.6 Fuel (Heating/Cooking) One-Way Sensitivity Graph

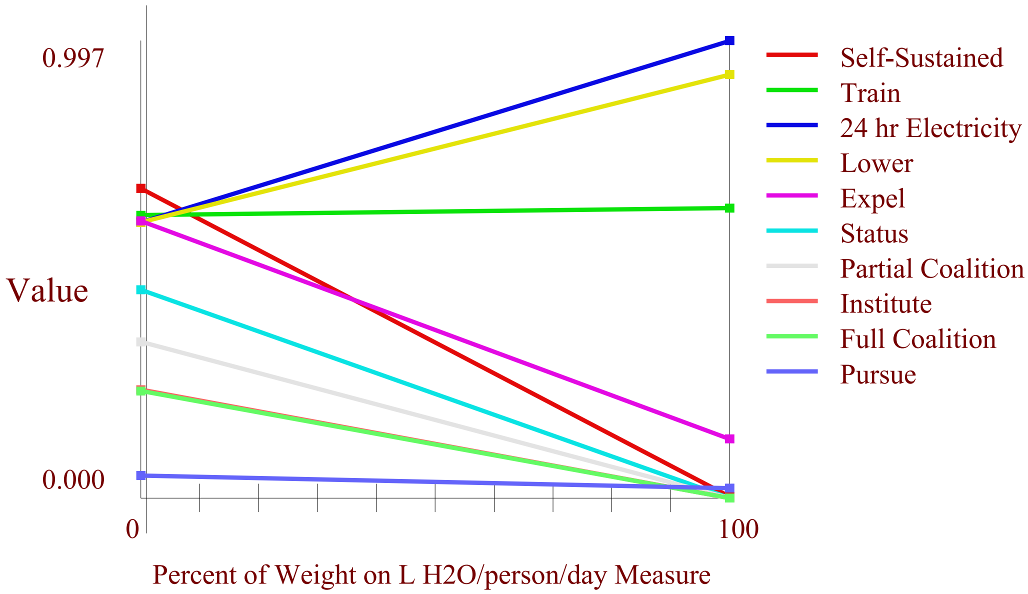


Figure C.7 Potable Water One-Way Sensitivity Graph

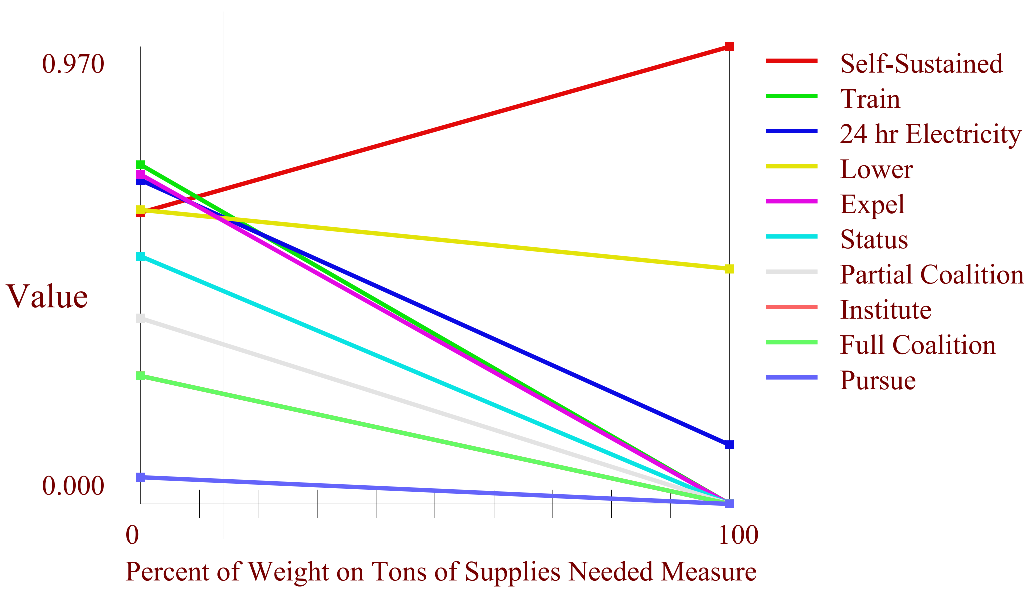


Figure C.8 Humanitarian Relief One-Way Sensitivity Graph

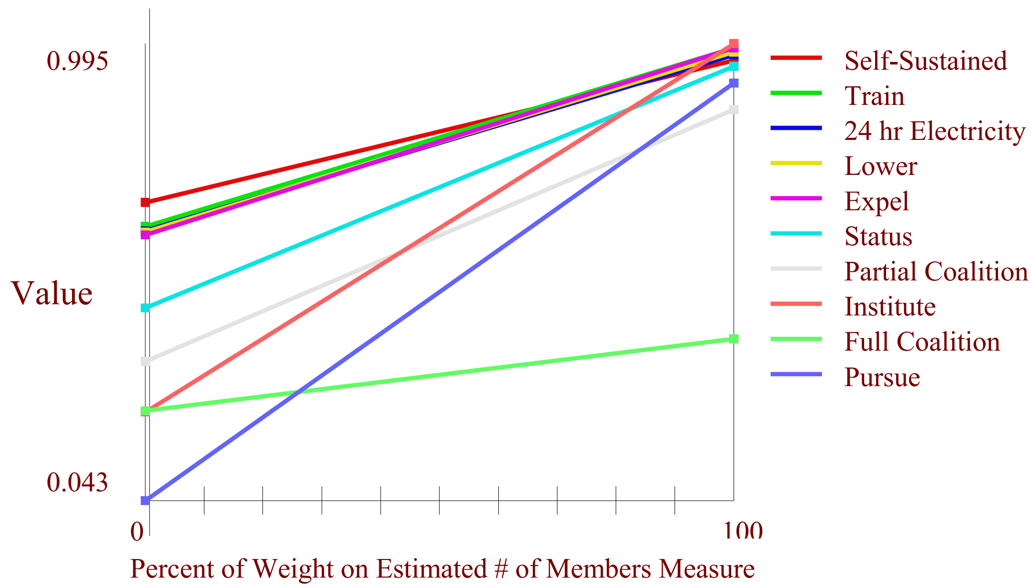


Figure C.9 Additional People Needed for ISI One-Way Sensitivity Graph

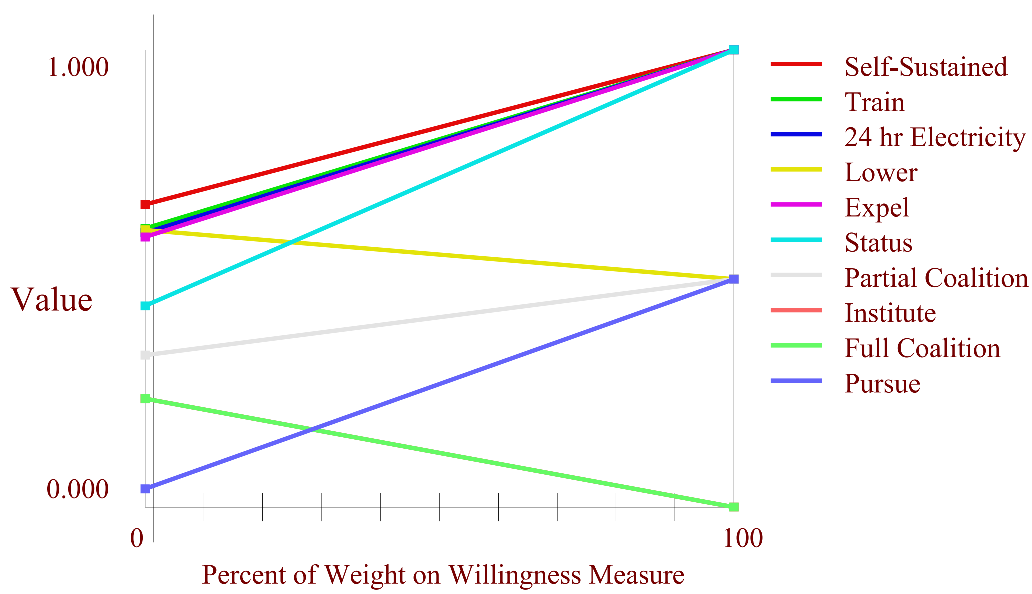


Figure C.10 Locals' Willingness to Serve One-Way Sensitivity Graph

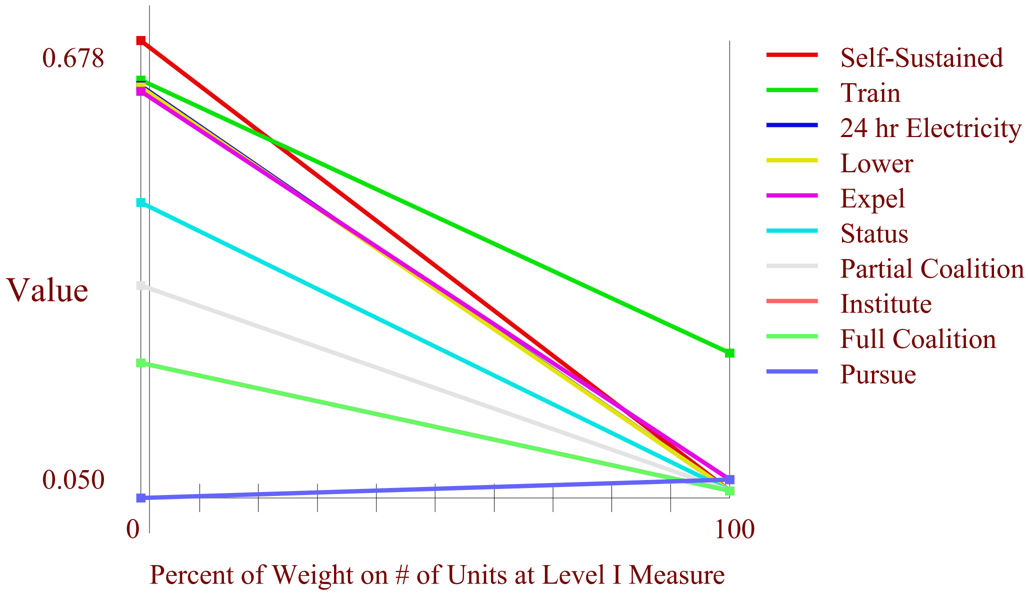


Figure C.11 Capability Level I One-Way Sensitivity Graph

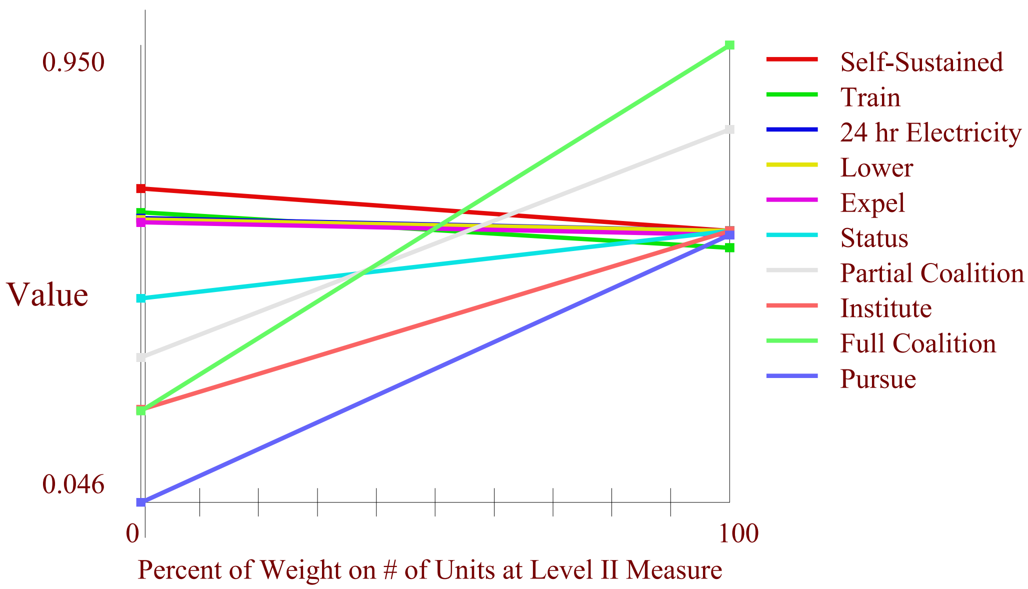


Figure C.12 Capability Level II One-Way Sensitivity Graph

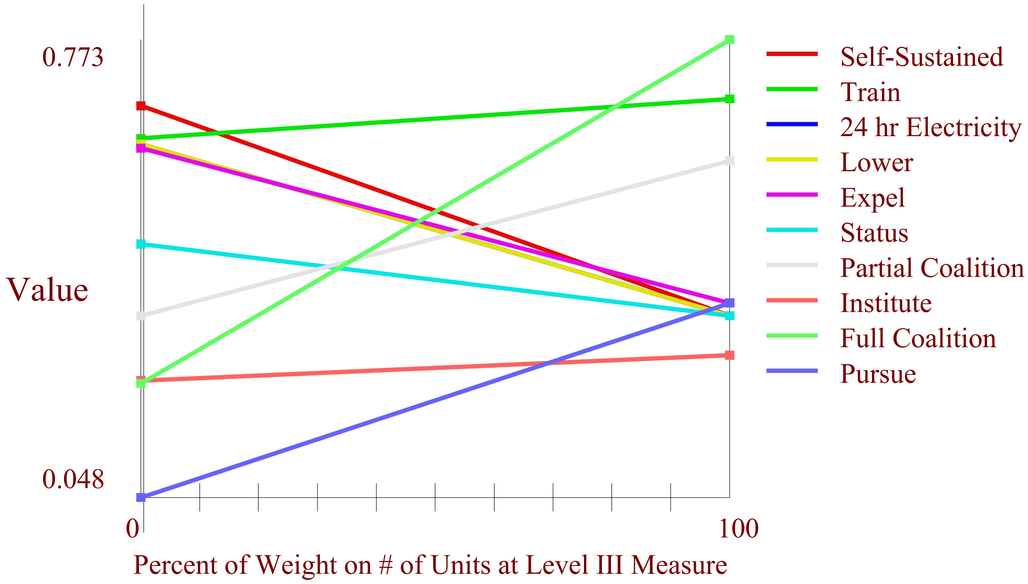


Figure C.13 Capability Level III One-Way Sensitivity Graph

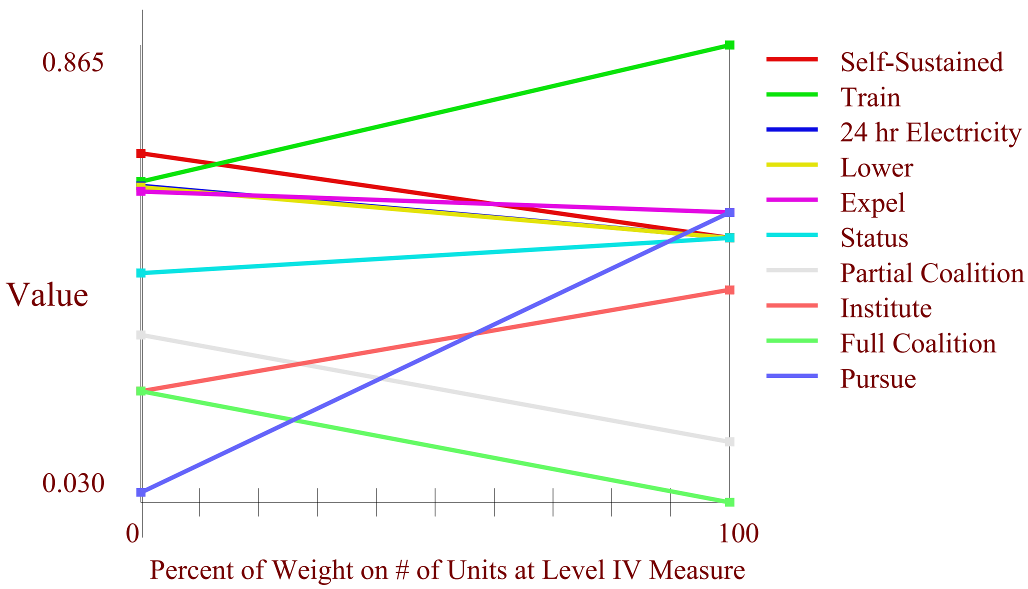


Figure C.14 Capability Level IV One-Way Sensitivity Graph

Appendix D. Robust Sensitivity Analysis of Attributes

A small change in a single weight using one-way sensitivity analysis indicates sensitive alternatives and the weight they are sensitive to. Looking at the robust sensitivity measures described in this research presents a new problem. Is it a large or small change in a weight needed for an alternative to become preferred that indicates sensitivity? The tables and diagrams below show a few approaches used to attempt to answer this question. Table D.1 shows the rank order of the absolute change in weight across each measure. Figure D.1 shows the information from Table D.1 as a contour plot breaking the ranks into the top, middle, and bottom five weights. Table D.2 looks at the rank order of the calculated weights for each measure compared to the rank order of the initial weights from the value model.

Table D.1 Rank Order of the Change in Weights for *Train ISI*

Train ISI	Least Squares	1-norm	∞-norm	2-norm	%
Tons of Supplies Needed	1	2	4	1	3
L H2O/person/day	2	1	3	2	8
# of Units at Level IV	3	3	1	3	13
# of Units at Level III	4	3	2	4	11
# of Units at Level II	5	3	8	5	9
# of Units at Level I	6	3	5	6	7
Willingness	7	3	12	7	6
Ethno-Sectarian Violence Death Rate	8	3	11	8	1
% Coalition Lost	9	3	6	9	2
Estimated # of Members	10	3	9	10	12
# Insurgents Crossing	11	3	13	11	5
Non-Sectarian Violent Death Rate	12	3	7	12	4
Avg. hrs./day	13	3	10	13	10
L Fuel/person/day	14	3	14	14	14

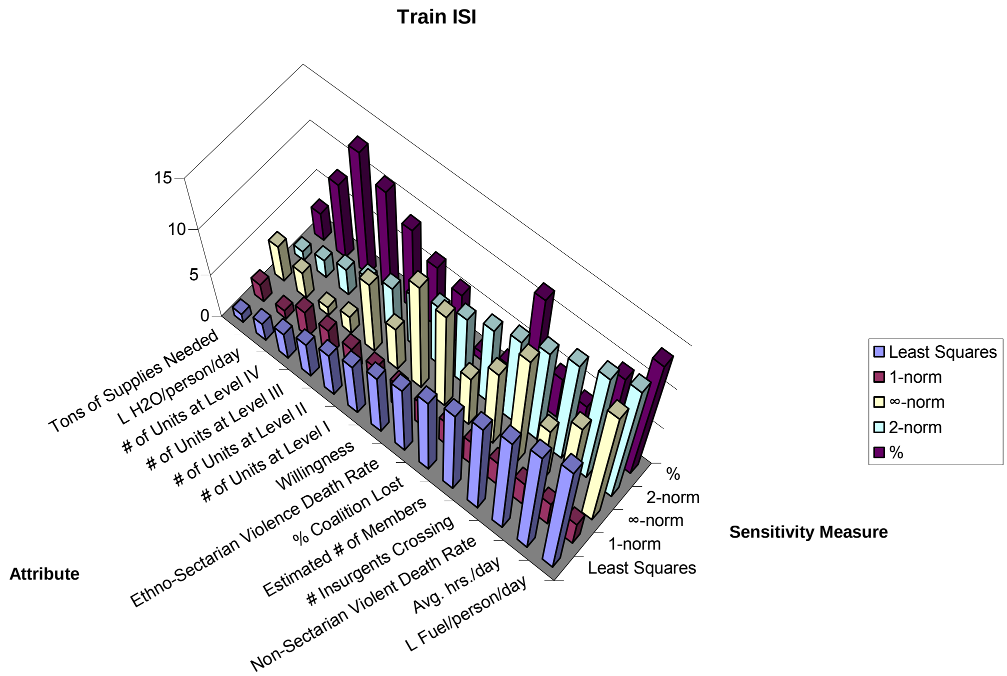


Figure D.1 3D Bar Graph of Data from Table D.1

Table D.2 Descending Rank Order of Weights for *Train ISI*

Train ISI	Swing Weighting	Least Squares	1-Norm	∞-Norm	2-Norm	%
Ethno-Sectarian Violence Death Rate	0.268	0.271	0.268	0.285	0.271	0.335
% Coalition Lost	0.215	0.212	0.215	0.193	0.212	0.161
Non-Sectarian Violent Death Rate	0.188	0.189	0.188	0.167	0.189	0.234
Tons of Supplies Needed	0.142	0.110	0.111	0.121	0.110	0.107
# Insurgents Crossing	0.107	0.108	0.107	0.093	0.108	0.082
Willingness	0.016	0.014	0.016	0.021	0.014	0.012
# of Units at Level I	0.016	0.020	0.016	0.034	0.020	0.020
L H2O/person/day	0.011	0.029	0.042	0.033	0.029	0.014
Estimated # of Members	0.008	0.006	0.008	0.000	0.006	0.007
# of Units at Level II	0.008	0.005	0.008	0.000	0.005	0.006
Avg. hrs./day	0.008	0.008	0.008	0.002	0.008	0.009
# of Units at Level III	0.005	0.014	0.005	0.027	0.014	0.007
L Fuel/person/day	0.004	0.004	0.004	0.001	0.004	0.004
# of Units at Level IV	0.003	0.011	0.003	0.024	0.011	0.003

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Vita

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14. ABSTRACT
The Department of Defense (DoD) requires the ability to quantifiably measure progress in arenas that are complex and difficult to measure, such as the stability of a region. Therefore, the DoD works diligently to predict the effect of operations and sponsors research to improve prediction and analysis. They desire a repeatable, systematic methodology to aid in the selection of courses of action (COA) that efficiently meet stated objectives and quantitatively measure the degree of accomplishment of these objectives. The author proposes a value-focused thinking (VFT) decision analysis (DA) approach to this problem. This methodology not only aids in selection of possible COAs, but provides a framework to compare the effectiveness of implemented actions via key indicators. Due to the complex nature of COA selection and assessment, weights within the DA model are often fluid. Sensitivity analysis provides the justification of COA selection in such an environment. This thesis focuses on conducting further analysis of the ranked alternatives through a robust sensitivity analysis technique.

Sensitivity analysis begins with the examination of the top ranked alternative by varying one weight at a time, one-way sensitivity. The author then proposes a more robust examination of multiple weight sensitivity using five unique measures and optimization via linear and non-linear programming. The measures reveal the alternatives sensitive to small simultaneous variations of multiple weights within the model, n-way sensitivity. Small measure values indicate sensitive alternatives, and indicate to a field commander where to more closely examine the consequences of a selected COA.

15. SUBJECT TERMS
Robust Sensitivity Analysis, Course of Action selection, n-Way Sensitivity Analysis, Affinity Diagramming, Additive Value Model, Stability Operations

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